2015

PHYSICS

[Honours]

PAPER – II (New)

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[NEW SYLLABUS]

GROUP – A

Answer any **two** questions : 15 × 2

1. (*a*) (*i*) Show that the force field given by $\vec{F} = -\alpha\vec{r}$, where $\vec{r}$ represents the position vector in $X-Y$ plane, is conservative. Here, α is a constant.

(Turn Over)

(*ii*) Find out the change in potential energy at two different points A and B for performing work.

(*iii*) Show that the angular momentum of a particle moving under the given force field remains constant. 2 + 3 + 2

(*b*) (*i*) A particle of mass m moves along the x-axis under the influence of a conservative force field due to the potential $V(x)$. Find out the time required for the particle in moving between the positions x_1 and x_2 is

$$\Delta t = \sqrt{\frac{m}{2}} \int_{x_1}^{x_2} \frac{dx}{\sqrt{E - V(x)}},$$

where E is the total energy.

(*ii*) From the standpoint of conservation of energy, derive the expression of position of the particle at time t for $V(x) = \beta(x - x_0)^2$, where β is a constant and x_0 is the position of zero potential energy. 2 + 4 (*c*) Distinguish between 'intertial'-mass and 'gravitational'-mass. 2

2. (*a*) In cylindrical coordinate system, velocity and acceleration are expressed usually as $\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{z}$ and $\vec{f} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + \left(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}\right)\hat{\phi} + \ddot{z}\hat{z}$ respectively. For a particle of mass m moving under the influence of a force $\vec{F} = \alpha\hat{\rho} + \vec{v} \times (\beta\hat{z})$, find out the physical entity which remains constant of motion. 5

(*b*) Prove that the total energy of a particle of mass m in motion under a central force field is given by

$$E = \frac{L^2}{2m}\left[\left(\frac{du}{d\theta}\right)^2 + u^2\right] + V(r),$$

where $V(r)$ is the potential energy and L is magnitude of angular momentum of the particle. $u = \frac{1}{r}$ and r, θ being the polar -coordinates of the particle. Hence, find out the differential equation for the orbit. 4 + 4 (*c*) Distinguish between external and internal torques in rigid body dynamics. 2

3. (*a*) The probability of a gas molecule having velocity component laying between u and $u + du$ in a definite direction is given by

$$f(u)\,du = a\,e^{-bu^2}\,du$$

where $a = \sqrt{\frac{m}{2\pi kT}}$ and $b = \frac{m}{2kT}$

(*i*) Use above formula to show the number of molecules having speed lying between c and $c + dc$ is given by

$$N(c)dc = 4\pi N a^3 e^{-bc^2} c^2 dc$$

where N is the total number of identical molecules in a vessel at temperature T.

(*ii*) Plot $N(c)$ *vs* c for two different gas molecules (say for hydrogen and oxygen). 2 + 2 (*b*) From the speed distribution formula find an expression for most probable speed (c_m) and r.m.s speed (c_{rms}). 2 + 2

(*c*) Show that the fraction of gas molecules whose velocities differ by less than 1·00 % from the value of the most probable speed (i.e. $c_m - 0{\cdot}01\ c_m < c < c_m + 0{\cdot}01\ c_m$ or $c \sim c_m < 0{\cdot}01\ c_m$) is about 1·66 %. 3

(*d*) Show that Joule-Thomson coefficient μ of a real gas is given by

$$\mu = \left(\frac{\partial T}{\partial p}\right)_H = -\frac{1}{C_p}\left[\left(\frac{\partial U}{\partial p}\right)_T + \left\{\frac{\partial}{\partial p}(pV)_T\right\}\right]$$

Interpret the appearance of two terms in the J-T coefficient. 3 + 1

4. (*a*) State and prove uniqueness theorem of electrostatics. 3

(*b*) A dielectric of permitivity $\in_2$ is placed in a uniform electric field $E_0\hat{z}$ in a medium of permitivity $\in$. (*i*) Show that the polarization induced in the sphere is

$$\vec{P} = 3\varepsilon_2\left(\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 - 2\varepsilon_2}\right)E_0\hat{z}.$$

(*ii*) Determine the electric field inside the sphere. 6

(*c*) Determine the magnetic vector potential $\vec{A}$ at a distance ρ from an infinitely long thin straight wire carrying current *I*. Hence find the corresponding magnetic field $\vec{B}$. 4

(*d*) A proton is moving, parallel to an infinitely long wire carrying current of 1 A, with velocity 0·6 c maintaining a separation of 10 cm. Calculate the force experienced by the proton. (Given, $\mu_0 = 4\pi \times 10^{-7}$ henry / m). 2

GROUP – B

Answer any **five** questions :

5. (*a*) A rocket starts falling vertically from rest under gravity. When it starts falling, the gas

ejected from the rocket at constant rate α maintains a speed v_0 w.r.t. the motion of the rocket. Determine the speed of the rocket and the distance travelled by it after time Δt. Given that M_0 is the initial mass of the rocket. 4 + 2

(*b*) Does the linear momentum in all collisions remain conserved ? 2

6. (*a*) For rotational motion of rigid bodies, derive an expression for kinetic energy in terms of moment of inertia and angular velocity. 3

(*b*) The three principal moments of inertia of a body at a point are given as 200, 300 and 450 gm-cm^2. Write down the equation of the ellipsoid of inertia at that point. What will be the kinetic energy and the angular momentum of the body when it is rotating about an axis passing through the point and having direction cosines $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}\right)$. The body is rotating at the rate 100 revolutions per minute. 3

(*c*) Show that the directions of angular velocity and angular momentum. through usually differ, coincide only along principal axes. 2

7. (*a*) Obtain the expression of gravitational self energy. 3

(*b*) Derive the expression of reduced mass in two-body problem. Why is it called so ? 3 + 2

8. (*a*) Assuming pressure(p) due to diffused black body radiation is one-third of the energy density of radiation (u) i.e. $p = 1/3\ u$, prove thermodynamically that $u = aT^4$; where 'a' is a constant and T is the absolute temperature of the black body. 4

(*b*) What is solar constant ? Assuming the sun as black body, estimate the temperature of the sun from the following data :

Angular diameter of the sun from the earth = 32 min

Solar constant = 1356 Wm^{-2}, Stefan constant, $\sigma = 5{\cdot}7 \times 10^{-8}\ Wm^{-2}K^{-4}$. 1 + 3

9. (*a*) Derive the equation of state of ideal gas by applying Virial theorem. 3

(*b*) Graphically discuss the discrepancy between theoretical and experimental curves based on the equation of state of van der Waals gas. 2

(*c*) From the given equation of state

$$P = \frac{RT}{V-b}\ e^{-\frac{a}{RTV}},$$

obtain the equation of state for van der Waals gas. All the physical parameters as mentioned here are of usual meaning. Mention the physical condition required for the derivation. 3

10. (*a*) A paint charge q is placed at a distance r from the center of a grounded conducting sphere of radius a. Calculate the surface charge density on the sphere for $r > a$. 5

(*b*) Obtain the expression of force per unit length between two long straight parallel conductors carrying current. 3

11. (*a*) Determine the potential at a point due to a static charge distribution in terms of various multipoles. 5

(*b*) Use the result of part (*a*) to determine the potential and hence the field due to a dipole. 3

12. (*a*) Starting from Ampere's law $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ obtain the modified form of it for a magnetized material

$$\oint \vec{H} \cdot d\vec{l} = I_f,$$

where the symbols are usual. 3

(*b*) Show that in a magnetized material where there is no free current $\vec{B}$ can be expressed as the gradient of a scalar potential which satisfies Laplace's equation. 2

(*c*) Find the boundary conditions on $\vec{B}$ and $\vec{H}$ at the interface of two magnetic media. 3

GROUP – C

Answer any **five** questions :

13. What is the effective length of a simple pendulum equivalent to a compound pendulum ? Show that

the centre of suspension and centre of oscillation of a compound pendulum are interchangeable w.r.t a certain condition. For two pair of points on two sides of the centre of gravity, the time period remains the same in case of a compound pendulum. Draw the diagram only in respect of this physical condition, so that it can be perceived. 1 + 1 + 2

14. Calculate the moment of inertia of a homogeneous right circular solid cone about its generating line (i.e. a straight line on its sloping side). 4

15. Show that the number of molecules striking unit area of a surface per unit time is $\frac{1}{4}n\bar{c}$, where $\bar{c}$ is the average speed and n is the number of molecules per unit volume. 4

16. Derive the expression of radial temperature distribution in a spherical shell at steady state. 4

17. Derive second latent heat equation with its physical significance. 3 + 1

18. (*a*) Three equal point charges $+q$ are located at the vertices of an equilateral triangle of sides 'a'. What charge must be placed at the centroid to make the total electrostatic energy zero ? 2

(*b*) Charges, each of magnitude $+q$, are placed on each vertex of a regular pentagon. Determine the electric field at the centre of the pentagon. 2

19. A dipole with moment p_1 is fixed at the origin of a co-ordinate system and another with moment p_2 is at the position vector $\vec{r}$ and is able to rotate about its centre. Show that, for equilibrium $\tan\theta_1 = 2\tan\theta_2$, where θ_1 and θ_2 are the angles that $\vec{r}$ makes with the first and second dipoles respectively. 4

20. Find the boundary conditions on $\vec{B}$ and $\vec{H}$ at the interface of two magnetic media. 4