# OLD

# 2015

## Part-I 3-Tier

## PHYSICS

### PAPER—I

### (Honours)

*Full Marks* : 90

*Time* : 4 Hours

*The figures in the right-hand margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Illustrate the answers wherever necessary.*

Answer any *two* questions from Group—A and *five* from each of the Group—B and C.

### Group—A

Answer any *two* questions.

**1.** (a) Find the first three non-zero terms in the Taylor's series for $ln(1+x)$ about $x = 0$. 3

(b) State Stokes' theorem. Give an example where this theorem is not valid. 2+2

(c) If $\vec{A} = ax\hat{i} + by\hat{j} + cz\hat{k}$ where a, b, c are constants, evaluate $\int_S \vec{A}.d\vec{s}$ where S is the surface of a sphere of radius R. 3

(d) Show that $\int_{-1}^{+1} P_n(x)\, P_m(x)\, dx = 0$ for $n \neq m$ ; $P_l(x)$ is the Legendre polynomial of order $l$. 3

(e) Write down the Gaussian distribution function. Draw the nature two distributions with same mean ($\mu$) but having different standard deviations ($\sigma$).

**2.** (a) Use the method of separation of variables to solve

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$$

Given that u (0, y) = 8 exp (–3y). 4

(b) What is meant by internal bending moment of a bent-beam ? Find its expression at a point with a radius of curvature R in terms of Young's modulus Y of its material and the geometrical moment of intetia I of its cross-section. 1+4

(c) What is an ideal fluid ? A fluid is at rest under an external force $\vec{f}$ per unit volume and pressure gradient force. If the pressure be $p$ at a point (x, y, z), show that at equilibrium $\vec{f} = \vec{\nabla} p$. 2+4

3. (a) Use Stokes' theorem to deduce

$$\int_{s} d\vec{s} \times \vec{\nabla}\varphi = \oint_{c} \varphi \, d\vec{r}$$

where $\varphi$ is a scalar. 3

(b) Prove the angular momentum theorem $\vec{N}^{(exL)} = \frac{d\vec{L}}{dt}$ for a system of particles mentioning the condition for its validity. 4+1

(c) A particle of mass *m* moves under the action of central force $\bar{F} = f(r)\bar{r}$. Show that the equation for the path of the particle is

$$\frac{d^2u}{d\theta^2} + u = -\frac{f(1/u)}{mL^2u^2}$$

Where L is a constant of motion and $u = 1/r$. 5

(d) Show that the path of the particle moving in the central force field must be a plane curve. 2

**4.** (a) A linear harmonic oscillator moving along x-axis has a mass 'm' and natural angular frequency $w_0$. It is subjected to a damping force $2bm\frac{dx}{dt}$ and is acted on by a periodic force $F_0 \cos \omega t$.

(i) Write down the equation of motion. 1

(ii) Prove that in the steady state, rate of supply of energy is equal to the rate of dissipation of energy. 4

(iii) Draw average power *vs.* frequency curve for two different damping coefficients $b_1$ and $b_2$ $(b_2 > b_1)$ in the same graph. 2

(b) What are Newtonian and non-Newtonian fluids ? Give examples of each. 2

(c) Legendre's equation has the form

$$(1 - x^2)\, y'' - 2x\, y' + l(l+1)\, y = 0,$$

where l is a constant.

Show that x = 0 is an ordinary point and x = ±1 are regular singular points of this equation. 2

(d) Prove the vector identity

$$\vec{\nabla} \times \left( \vec{\nabla} \times \vec{A} \right) = \vec{\nabla} \left( \vec{\nabla} . \vec{A} \right) - \nabla^2 \vec{A} .$$ 4

**Group—B**

5. (a) (i) Find the trace and the determinant of the matrix

$$A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} .$$

Find the eigenvalues of A. 2+2

(ii) If a matrix A is Hermitian and $A^2 = I$, show that A is also unitary. 2

(b) A random variable x has a probability function f(x). Define variance and standard deviation of x. 2

6. (a) Laplace's equation in spherical polar co-ordinates (with azimuthal symmetry) is given by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v}{\partial\theta}\right)=0.$$

(i) Assume separation of variable solution

$$v = R(r)\ (H)\ (\theta).$$

To show that the Laplace's equation can be decoupled into two total differential equations. 1

(ii) Assume the separation constant to be of the form $l(l+1)$, show that the solution of R is

$$R(r) = A\ r^l + B\ r^{-(l+1)}.$$ 1+2

(b) If the solution of Hermite's differential equation

$$\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2xy=0$$ is written as $$y=\sum_{r=0}^{\infty}a_r x^{k+r},$$

then show that the allowed values of k are 0 and 1 only. 4

7. (a) A frame of reference rotates with angular velocity $\vec{\omega}$. For this frame establish the identity

$$\frac{d}{dt} = \frac{d'}{dt} + \vec{\omega} \times .$$

Where $\frac{d}{dt}$ and $\frac{d'}{dt}$ stand for time derivative w.r.t fixed and rotating frame respectively. 4

(b) For a thin, uniform, square plate of side 'a' and mass m, what are the principal axes of inertia ? Derive the principal moments of inertia. 1+3

8. (a) What is group velocity ? Derive its relationship with phase velocity. The phase velocity in a medium (i) increases with frequency, (ii) decreases with frequency. In which case will the group velocity be greater than the phase velocity ? 1+3+1

(b) Consider the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Show that $y(x, t) = f(kx - \omega t)$ is a solution of the wave equation where $\omega = kv$. 3

9. (a) Show that the kinetic energy of a system of particles is equal to the kinetic energy of a single particle of total mass M situated at the centre of mass, together with kinetic energy of the system of particles with their motion relative to the centre of mass. 5

(b) Show that the force field $\vec{F}$ defined by

$$\vec{F} = \left(y^2z^3 - 6xz^2\right)\hat{i} + 2xyz^3\hat{j} + \left(3xy^2z^2 - 6x^2z\right)\hat{k}$$

is a conservative force field. 3

10. (a) Define surface tension. A spherical soap bubble is slowly enlarged from a radius of 1 cm to a radius of 10 cm. The surfae tension of the soap solution is $0{\cdot}026\ \text{Nm}^{-1}$. Calculate the work done in the process. 1+3

(b) Prove that for a light cantilever of length $l$ and carrying a weight W at its free end, the depression at the free end is

$$\delta = \frac{W\,l^3}{3YI},$$ I being the geometrical moment of inertia and Y the Young's modulus of the material of the cantilever. 4

11. (a) Legendre polynomial may be expressed as

$$\left(1-2xt+t^2\right)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} P_n(x)t^n$$

Use this to show that $P_n(1) = 1$. 2

(b) State clearly the assumptions made to derive Poiseuille's formula for flow of liquid through a narrow tube.

(c) A liquid of coefficient of viscosity $\eta$ flows steadily through a cylindrical tube of radius 'a' and length '$l$' under a pressure 'P'. Show that its velocity at a point inside the tube at a distance r from its axis is

$$v = \frac{P}{4\eta l}\left(a^2 - r^2\right).$$ 4

12. (a) For stationary waves the displacement of a point at x at time t in case of transverse vibration of a stretched string under tension T and length $l$, is given by

$$y(x, t) = \sum_{s=1}^{\infty} \frac{A}{s^2} \operatorname{Sin}\frac{s\pi a}{l} \operatorname{Sin}\frac{s\pi x}{l} \operatorname{Cos}\frac{s\pi ct}{l}$$

where $x = a$ is the point of excitation and c is the velocity of the transverse wave along the string.

(i) Find the initial displacement for the 3rd harmonic. 1

(ii) If the string is excited at $x = a = l/2$, give the harmonics which will be absent. 2

(iii) What happens if the string after excitation be touched at the same point $x = a$? 2

(b) A source of sound emits energy equally in all directions at the rate of 0·5 J/s. What is the intensity level at a distance of 10m from the source? Take the threshold level for intensity as $10^{-12}$ W/m$^2$. 3

## Group—C

Answer any *five* questions.

**13.** State Kepler's laws for planetary motion. 4

**14.** Find out the gravitational intensity due to a solid uniform sphere inside the sphere. 4

**15.** Using principle of dimensional homogeneity deduce stokes law, relating to viscous force. 4

16. Suppose $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be the position vector of a point in the space. Consider cylindrical polar co-ordinates $(\rho, \varphi, z)$.

(i) Express x, y, z in terms of $\rho$, $\varphi$, z. Find the unit vectors $\hat{e}_\rho$, $\hat{e}_\varphi$ and $\hat{e}_z$.

(ii) Prove that the cylindrical polar co-ordinate system is orthogonal. 2+2

17. Expand the function

$f(x) = z$ for $-\pi < x < \pi$

$f(x + 2\pi) = f(x)$

in a Fourier series. 4

18. Solve the equation

$$\frac{d^2x}{dt^2} + b^2x = k \cos bt$$

subject to the conditions $x = 0$ and $\frac{dx}{dt} = 0$ at $t = 0$.

4

**19.** A particle is simultaneously under the action two simple harmonic motions at right angles to each other, represented by $x = a \sin \omega t$, $y = b \sin(\omega t + \delta)$.

Show that the resultant motion is represented by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos\delta = \sin^2\delta .$$

What will be the locus of the particle when $\delta = \pi/2$ and $a = b$? 3+1

**20.** (a) What are reverberation and reverberation time? 2

(b) Distinguish between 'bel' and 'phon'. 2

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