UG/I/PHS/H/I/15

2015

PHYSICS

[Honours]

PAPER – I (New)

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[NEW SYLLABUS]

GROUP – A

Answer any **two** questions

1. (*a*) Find the Fourier expansion of the function :

$$f(x) = \begin{cases} 0, \text{ for } -\pi < x < 0 \\ 1, \text{ for } 0 < x < \pi \end{cases} \qquad 3$$

Hence show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ 1

(*b*) Evaluate $\oint_C \vec{A}\cdot d\vec{r}$ and $\iint_S (\vec{\nabla}\times\vec{A})\cdot d\vec{S}$, where $\vec{A} = (2x-y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. Hence comment whether Stokes' theorem valid in the above case. $2 + 2\frac{1}{2} + \frac{1}{2}$

(*c*) General expression of curl in orthogonal curvilinear coordinates is given by

$$\vec{\nabla}\times\vec{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_1} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix},$$

where the symbols have their usual meaning. Express Curl $\vec{A}$ in terms of spherical polar coordinates (expression may be kept in determinant form). 4

(*d*) Define axial vector with example. 2

2. (*a*) What do you mean by optical activity of a crystal and of a solution ? On what factors do they depend ? 2 + 2

(*b*) Define angle of polarisation. Light of wavelength 560 nm gets plane polarised when reflected at an angle 60° from the plane surface of a glass. What is the refractive index of this glass ? What is the angle of refraction of the refracted beam ? 2 + 1 + 1

(*c*) What is quarter wave plate ? Give the principle of production of circularly polarised light from plane polarised light by quarter wave plate. 2 + 3

(*d*) A 2 cm wide diffraction grating is just able to resolve Na D lines in 2nd order. Find the number of rulings per mm. 2

3. (*a*) Find the velocity, frequency and wavelength of the plane progressive wave given by : $\psi = 5 \cos (640\pi t - 2\pi x)$ meter. 3

(*b*) What do you mean by group velocity ? 1

(*c*) Two plane progressive waves of equal amplitude and slightly different frequencies ω and $\omega + d\omega$ travelling along same direction

with propagation constants k and $k + dk$ combine to form a wave group. Show that the phase velocity and group velocity of the wave group are given by :

$$v_p = \frac{\omega}{k} \text{ and } v_g = \frac{d\omega}{dk}.$$ 4

(*d*) The general expression of displacement of a string stretched between two fixed support is given by :

$$y(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{l}\left(a_n \cos\frac{n\pi ct}{l} + b_n \sin\frac{n\pi ct}{l}\right),$$

where the symbols have usual meaning.

(*e*) Use this to obtain the expression of $y(x, t)$ for a struck string. 5

(*f*) Show that all the harmonics having a node at the point of striking are absent. 1

(*g*) What is the binary equivalent of 25 ? 1

4. (*a*) Show that the elastic potential energy per unit volume due to shear can be expressed as : $U = \frac{1}{2} \times$ Shearing Stress $\times$ Shearing Strain. 2

(*b*) Obtain the expression of depression of the free end of a light cantilever due to load applied at that end. Hence deduce the working formula for determination of Young's modulus of the material of a beam by flexure method. 3 + 2

(*c*) Explain 'interference by division of wave front' and 'interference by division of amplitude' with examples. Which of these requires narrow source and which requires broad source ? $\left(1\frac{1}{2}+1\frac{1}{2}\right)+1$

(*d*) Using OR, AND and NOT gates draw a circuit for obtaining the following truth table : 4

Input		Output
A	*B*	*Y*
0	0	0
0	1	1
1	0	1
1	1	0

GROUP – B

Answer any **five** questions

5. (*a*) Verify whether the vector

$$\vec{F} = \frac{-y\hat{i} + x\hat{j}}{\sqrt{x^2 + y^2}}$$

is conservative or solenoidal or both. 3

(*b*) Obtain the formal Taylor series of $\ln(1 + x)$ about $x = 0$. 2

(*c*) Solve the differential equation : 3

$$y'' + 2y' + y = e^{-ix}.$$

6. (*a*) Solve by Frobenius method : 4

$$4x^2y'' + 4xy' + (4x^2 - 1) = 0.$$

(*b*) Assuming azimuthal symmetry about z-axis write down the solution of Laplace's equation in spherical polar coordinates. 1

(*c*) In a problem of azimuthal symmetry the potential function on z-axis is given by : 3

$$V_{\text{axis}} = \left(1 + \frac{z^2}{a^2}\right)^{1/2} - \frac{z}{a};$$

Using the solution of Laplace's equation find the potential at any point (r, θ, ϕ) in the region where $z < a$.

[Given : for $x < 1, (1+x)^{1/2} = 1 + \frac{1}{2}x^2 - \frac{1}{2.4}x^4 + \frac{1.3}{2.4.6}x^6 - \cdots$]

7. The expression of velocity in a forced vibration is given by :

$$v = \frac{F\cos(\omega t - \alpha)}{\left[k^2 + \left(m\omega - \frac{s}{\omega}\right)^2\right]^{1/2}}, \text{ where } \tan\alpha = \left(m\omega - \frac{s}{\omega}\right)/k$$

and other symbols have usual meaning.

(*a*) Show that average power in forced vibration is given by : $P_{av} = F_{rms} v_{rms} \cos\alpha$. 4

(*b*) What is power factor ? 1

(*c*) Show that at velocity resonance power factor equals to unity. 1

(*d*) Which of the Ramsden's eyepiece and Huygen's eyepiece is used in travelling vernier microscopes and spectrometer telescopes and why ? 2

8. (*a*) Obtain the expression of work done against surface tension to divide a liquid drop of radius R into N number of identical droplets. 3

(*b*) A U-tube contains a liquid of surface tension S and is kept with its arms vertical. Two arms of the U-tube have radii R_1 and R_2. If the liquid has zero angle of contact with the walls of the tube then obtain the expression of difference of its height in the two arms of the U-tube. 3

(*c*) Define dispersive power of a prism. 2

9. (*a*) State Fermat's principle and obtain laws of reflection of light at a plane surface from it. 1 + 3

(*b*) Consider a plano-convex lens of a material of refractive index 1·5. The convex surface has a radius of curvature 2·5 cm and is facing the incident light. The centre thickness of the lens is 0·6 cm. Construct the system matrix. 4

10. (*a*) Let $\vec{A} = \phi \vec{C}$, where $\vec{C}$ is a constant vector. Use Stokes' theorem to deduce that

$$\int_S d\vec{s} \times \vec{\nabla}\phi = \oint_C \phi \, d\vec{r}. \qquad 3$$

(*b*) State clearly the assumptions made to derive Poiseuille's formula for flow of liquid through a narrow tube. 2

(*c*) A liquid of coefficient of viscosity η flows steadily through a cylindrical tube of radius a and length l under a pressure difference P. Show that its velocity at a point inside the tube at a distance r from the axis is

$$v = \frac{P}{4\eta l}(a^2 - r^2).$$ 3

11. (*a*) Explain how depletion region is formed at the *p-n* junction of a semiconductor diode. How does it get widened with the application of reverse bias voltage ? 2 + 2

(*b*) Draw the circuit diagram of a bridge rectifier with a π filter connected to it. 3

(*c*) When a Zener diode can be used as a rectifier diode ? 1

12. (*a*) A particle is simultaneously under two simple harmonic motions at right angles to each other, represented by $x = a \sin wt$, $y = b \sin (wt + \delta)$.

(*i*) Show that the resultant motion is represented by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cdot\cos\delta = \sin^2\delta. \qquad 3$$

(*ii*) What will be the locus of the particle when $\delta = \pi/2$ and $a = b$? 2

(*b*) An achromatic converging combination of mean focal length 60 cm is formed with a convex lens of crown glass and a concave lens of flint glass placed in contact with each other. The dispersive power of crown glass is 0·03 and that of flint glass is 0·05. Calculate the focal lengths of the components. 3

GROUP – C

Answer any **five** questions

13. The generating function for Legendre polynomials is given by :

$$\left(1 - 2xt + t^2\right)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)\, t^n.$$

Prove :

(*a*) $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$. 3

(*b*) $P_n(-x) = (-1)^n P_n(x)$. 1

14. Verify $\overline{A+B} = \overline{A}.\overline{B}$ by a truth table and draw the required circuit using logic gates for verification of this relation. 2 + 2

15. In a Fraunhofer double slit diffraction pattern the angular separation between the 1st oder and zero order diffraction maxima is 17·19′ and the 4th order diffraction maxima is missing for a light of wavelength 500 nm. Find the width of each slit and the opaque space between them. 4

16. Calculate the area, inner radius and outer radius of the 10th half period zone for a plane wave front of light of wave length 500 nm with respect to a point at a distance 0·5 m from it. 4

17. Suppose $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be the position vector of a point in the space. Consider cylindrical polar co-ordinates (ρ, ϕ, z). Express x, y, z in terms of ρ, ϕ, z.

(*a*) Find the unit vectors $\hat{e}_\rho, \hat{e}_\phi$, and $\hat{e}_z$. 2

(*b*) Prove that the cylindrical polar co-ordinate system is orthogonal. 2

18. (*a*) What are the cardinal points of a thick lens ? 2

(*b*) Define principal points in a thick lens. 2

19. (*a*) A hall of volume 2500 m^3 has a total absorption equivalent to 300 m^2 of open window. There are 10 persons in the room, each equivalent to 4 sabin absorption. What is the reverberation time of the room ? 2

(*b*) Draw the circuit diagram for CE output characteristics of an *n-p-n* transistor. 2

20. Two cylindrical shafts have the same length and mass and are made of the same material. One is solid while the other is hollow. The external radius of the hollow cylinder is twice the internal radius. Find out the ratios of their torsional rigidities. 4
