UG/III/MATH/H/VI/15

2015

MATHEMATICS

[Honours]

PAPER – VI

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[NEW SYLLABUS]

GROUP – A

(*Analytical Statics*)

[*Marks* : 36]

1. Answer any *one* question : 12 × 1

(*a*) A body rests in equilibrium upon another fixed body, the portions of the two bodies in contact being spheres of radii r and R

respectively, and the straight line joining the centres of the spheres being vertical. If the first body be slightly displaced, find whether the equilibrium is stable or unstable, the bodies being rough enough to prevent sliding. 12

(*b*) (*i*) Three forces act along the straight lines $x = 0,\ y - z = a$; $y = 0,\ z - x = a$; $z = 0,\ x - y = a$. Show that they cannot be reduced to a couple. Prove also that if the system reduces to a single force, its line of action must lie on the surface $x^2 + y^2 + z^2 - 2yz - 2zx - 2xy = a^2$. 8

(*ii*) Define "limiting friction" and "angle of friction" and write down their respective dimensions. 4

2. Answer any *two* questions : 8 × 2

(*a*) If any force P acts at a point A on a rigid body then it can be replaced by an equal and parallel force P acting at an arbitrary point 0 in the plane of the force on the body together with a couple whose moment is the moment of the force P about 0. 8

(*b*) A circular disc of radius '*a*', whose density is proportional to the distance from the centre, has a hole cut in it bounded by a circle of diameter '*b*', which passes through the centre. Show that the distance from the centre of the disc of the Centre of Gravity of the remaining portion is $\frac{6b^4}{15\pi a^3 - 10b^3}$. 8

(*c*) A solid hemisphere of weight W rests in limiting equilibrium with its curved surface on a rough inclined plane and its plane face is kept horizontal by a weight P attached to a point in the rim. Prove that the coefficient of friction is $\frac{P}{\sqrt{W(2P+W)}}$. 8

(*d*) A small ring is carried on a smooth wire bent into the form of a curve such that the sum of the cosines of the base angles is constant. Find the relation between two forces directed towards the pole if they keep the ring in equilibrium. 8

3. Answer any *two* questions : 4×2

(*a*) When is a system of Coplanar forces acting on a rigid body said to be in astatic equilibrium ? Find the condition for a given system of coplanar forces to be astatic equilibrium. 4

(*b*) Show that every given system of forces acting on a rigid body can be reduced to a wrench. 4

(*c*) Four equal heavy uniform rods, each of weight w are freely jointed to form a rhombus $ABCD$ and is suspended by the joint A. A light rod connects the middle points of AB and AD so that the rhombus cannot collapse. Prove that the thrust of the stiffening rod is $4w\tan\alpha$, where the angle BAC is α. 4

GROUP – B

(*Hydrostatics*)

[*Marks* : 27]

4. Answer any *one* question : 9×1

(*a*) Deduce the formula $HM = \frac{AK^2}{v}$, the symbols have usual meanings. Find the meta centric height of a ship for rolling displacements. $6 + 3$

(*b*) If the absolute temperature T diminishes upwards in the atmosphere according to the law $T = T_0/(1 + \alpha z)$, where α is constant, show that the pressure at a height z is given by

$$p = p_0 \exp\left\{-\left(\frac{z}{H} + \frac{\alpha z^2}{2H}\right)\right\},$$

where p_0 is the pressure and T_0 is the temperature at $z = 0$ and H is the height of the homogeneous atmosphere. 9

5. Answer any *two* questions : 6×2

(*a*) Deduce a necessary and sufficient condition of equilibrium for maintaining a fluid under a given system of forces. 6

(*b*) ABC is a triangular lamina with the side AB in the surface of a heavy homogeneous liquid. A point D is taken in AC such that the thrusts on the areas ABD and DBC are equal. Prove that $AD : AC = 1 : \sqrt{2}$. 6

(*c*) A circular disc of radius 'a' is just completely immersed with its plane vertical in a homogeneous liquid. Prove that the distance between the centres of pressure of the two semi-circles into which the disc is divided by its horizontal diameter is $\frac{18\pi a}{9\pi^2 - 16}$. 6

(*d*) Three liquids whose densities $\rho_1, \rho_2, \rho_3 (\rho_1 > \rho_2 > \rho_3)$ are in A.P. completely fill a circular tube in vertical plane and occupy lengths of arcs subtending angle 2α, 2β, 2γ, at the centre, if the radius to the point midway between the ends of the lightest liquids makes with the vertical angle θ, prove that $\cot\theta = \cot\alpha + 2\cot\beta$. 6

6. Answer any *two* questions : 3 × 2

(*a*) Show that the thrust of a heavy homogeneous liquid on a plane area completely immersed in it is equal to the product of the area and the pressure at its centre of gravity. 3

(*b*) Define the surface of floatation and surface of buoyancy. 3

(*c*) Show that the equation for a gas in an adiabatic temperature change is

$$Tv^{\gamma-1} = \text{constant},$$

the symbols having usual meaning. 3

GROUP – C

(*Rigid Dynamics*)

[*Marks* : 27]

7. Answer any *one* question : 9 × 1

(*a*) Show that the equation of the momental ellipsoid at a point on the circular edge of a solid cone of height h

$$(3a^2 + 2h^2)x^2 + (23a^2 + 2h^2)y^2 + 26a^2z^2 - 10ahxz = \text{constant},$$

'a' being the radius of the base of the cone. 9

(*b*) State D' Alembert's principle and deduce general equations of motion of a rigid body. Prove that the motion of a rigid body about its centre of inertia is the same as it would be of the centre of inertia were fixed and the same forces acted on the body. 1 + 4 + 4

8. Answer any *two* questions : 6 × 2

(*a*) Find the moment of inertia of the area of the curve $r^2 = a^2 \cos 2\theta$ about a straight line through the origin and perpendicular to its plane. 6

(*b*) Show that the rate of change of angular momentum of a rigid body about the axis of rotation is equal to the sum of moments about the same axis of all forces acting on the body. 6

(*c*) A vessel in the form of a right cone with vertex downwards is filled with liquid and revolve with uniform angular velocity w about the axis ; if h be the height and 2α the vertical angle of the cone, show that the

amount of liquid that is split is

$\frac{1}{4}\frac{\pi w^2 h^4}{g}\tan^4\alpha$, provided that w is not greater than $\sqrt{\frac{2g}{h}}\cot\alpha$. 6

(*d*) A uniform rod AB, of mass m and length $2a$, is at rest. It is struck by a blow of impulse P at right angles to its length at a distance c from its middle point. Find the point about which it will begin to turn. 6

9. Answer any *two* questions : 3 × 2

(*a*) Deduce the principle of conservation of moment of momentum for finite forces. 3

(*b*) Deduce the minimum time of oscillation of a compound pendulum. 3

(*c*) Find the kinetic energy of a rigid body rotating about a fixed axis. 3