Total Pages—10

UG/II/MATH/H/III/15

2015

MATHEMATICS

[Honours]

PAPER – III

Full Marks : 90

Time : 4 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

(*Vector Analysis*)

[*Marks* : 27]

1. Answer any *one* question from the following : 8 × 1

(*a*) (*i*) Show that the vector $\frac{\vec{r}}{r^3}$ where

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

is both irrotational and solenoidal. 4

(*ii*) Find interms of k, the shortest distance between the lines $\vec{r} = \vec{\alpha} + t\vec{\beta}$ and $\vec{r} = \vec{\gamma} + s\vec{\delta}$ (t, s are scalars) where $\vec{\alpha} = (1,2,3)$, $\vec{\beta} = (2,3,4)$, $\vec{\gamma} = (k,3,4)$, and $\vec{\delta} = (3,4,5)$. For what value of k are the lines coplanar ? 4

(*b*) (*i*) Find the angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. 4

(*ii*) Show by vector method that the equation of the plane passing through the three points $(0, 0, 0)$, $(2, 4, 1)$ and $(4, 0, 2)$ is $x - 2z = 0$. 4

2. Answer any *four* questions : 4 × 4

(*a*) A particle moves according to the law $\vec{r} = \cos t\hat{i} + \sin t\hat{j} + t^2\hat{k}$. Find the velocity at any time t. Also, determine the tangential and normal components of acceleration of the particle. 4

(*b*) Let f and g be scalar functions. Prove that

$$\nabla\cdot(f\nabla g)-\nabla\cdot(g\nabla f)=f\nabla^2 g-g\nabla^2 f$$ 4

(*c*) Show that the line integral

$$\int_P^Q x^2 y dx + xyz dy + y^3 dz$$

is not independent of the path of integration, where P and Q are respectively (0, 0, 0) and (1, 1, 1). 4

(*d*) Verify divergence theorem for the vector functions $2xz\hat{i}+y^2\hat{j}+yz\hat{k}$ taken over the surface of the cube bounded by $x=0, x=1$, $y=0, y=1$ and $z=0, z=1$. 4

(*e*) Prove that

$$\int_S (y^2z^2\hat{i}+z^2x^2\hat{j}+x^2y^2\hat{k}).\hat{n}dS=\frac{\pi}{12}$$

where S is a part of the sphere $x^2+y^2+z^2=1$ above the xy plane and founded by this plane. 4

(*f*) Prove that the perpendiculars from the vertices of a triangle to the opporite sides meet at a point. 4

3. Answer any *one* question : 3 × 1

(*a*) What do you mean by conservative force ? Test whether the force

$$\vec{F} = (x^2 + 2yz)\hat{i} + (x - 2xyz)\hat{j} + (yz + z^2)\hat{k}$$

is conservative or not ? 1 + 2

(*b*) Define harmonic function. Show that the function $\phi(x, y, z) = x^2 - y^2 + 4z$ is harmonic. 1 + 2

GROUP – B

(*Analytical Geometry*)

[*Marks* : 45]

(*Analytical Geometry of two Dimensions*)

[*Marks* : 18]

4. Answer any *two* questions : 8 × 2

(*a*) Define ortho centre of a triangle. Show that the distance from the origin to the ortho centre of the triangle formed by the straight lines $\frac{x}{\alpha}+\frac{y}{\beta}=1$ and $ax^2+2hxy+by^2=0$ is $\alpha\beta(a+b)(\alpha^2+\beta^2)^{1/2}/(a\alpha^2-2h\alpha\beta+b\beta^2)$. 8

(*b*) If *A* and *B* be two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ such that three times the ecentric angle of the one is equal to the supplement of that of the other, then find the locus of the pole of AB with respect to the ellipse. 8

(*c*) Find the equation of the chord joining the points $\theta=\alpha-\beta$ and $\theta=\alpha-\beta$ of the conic $\frac{l}{\pi}=1-e\cos\theta$. Hence find the equation of the tangent to the conic at the $\theta=\alpha$. 8

5. Answer any *one* question : 2 × 1

(*a*) Show that the equation

$$11x^2-4xy+14y^2-58x-44y+71=0$$

represents a cental conic. 2

(*b*) If $ax + by$ transforms to $a'x' + b'y'$ due to rotation of axes, then show that

$$a^2 + b^2 = a'^2 + b'^2. \qquad 2$$

(*Analytical Geometry of three Dimensions*)

[*Marks* : 27]

6. Answer any *one* question : 15 × 1

(*a*) (*i*) Reduce the equation

$$6y^2 - 18yz - 6zx + 9x + 5y - 5z + 2 = 0$$

to the canonical form and state the nature of the surface represented by it. 8

(*ii*) Prove that the planes $x = ry + qz$, $y = pz + rx$, $z = qx + py$ pass through one line if $p^2 + q^2 + r^2 + 2pqr = 1$ and show that the equations of the lines are

$$\frac{x}{\sqrt{1-p^2}} = \frac{x}{\sqrt{1-q^2}} = \frac{x}{\sqrt{1-r^2}} \qquad 7$$

(*b*) (*i*) Find the length of the perpendicular, drawn from origin to the line $x + 2y + 3z + 4 = 0 = 2x + 3y + 4z + 5$. Find the equation of this perpendicular and the co-ordinates of the foot of the perpendicular. 7

(*ii*) Show that the perpendicular from the origin on the generators of the paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{c}$ lie on the cone

$$\left(\frac{x}{a} - \frac{y}{b}\right)(ax - by) + 2z^2 = 0.$$ 8

7. Answer any *one* question : 8 × 1

(*a*) Define circle in 3-dimension P is a variable point on a given line and A, B, C are its projections of the axis. Show that the sphere OABC passes through a fixed circle. 8

(*b*) If the equation

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

represents a right circular cone of semi vertical angle α, then prove that

$$\frac{gh}{f}-a=\frac{hf}{g}-b=\frac{fg}{h}-c=\left(\frac{gh}{f}+\frac{hf}{g}+\frac{fg}{h}\right)\cos^2\alpha$$ 8

8. Answer any *one* questions : 4 × 1

(*a*) Find the equation to the right circular cylinder whose radius is 5 and whose axis is

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$ 4

(*b*) Find the equation to the surface obtained by revolving the line $y=0$, $z=-2$ about the x-axis. 4

GROUP – C

(*Astromy*)

[*Marks* : 18]

9. Answer any *one* questions : 15 × 1

(*a*) (*i*) If z_1 and z_2 be the zenith distances of a star on the meridian and on the prime vertical respectively, then prove that

$$\sin z_1 = (\sec z_2 - \cos z_1)\tan\delta$$ 7

(*ii*) Establish the formula

$$\tan\frac{PSQ}{2} = \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a+b}},$$

where P and Q are two planets at a distance a and b respectively from the sum S. 8

(*b*) (*i*) Establish the effect of parallax on the longitude and latitude of a star. Also, show that on account of parallax a star describes an ellipse in course of the year. 8

(*ii*) Deduce the Cassini's formula for astronomical refraction in the form $R = A\tan z + B\tan^3 z$, where A and B are constants and z is the zenith distance of the body. 7

9. Answer any *one* questions : 3 × 1

(*a*) Show that the aberration varies as the sine of the earth's way. 3

(*b*) Show that the altitude of a heavenly body is greatest when on the meridian. 3
