UG/III/MATH/H/VI/16

2016

MATHEMATICS

[Honours]

PAPER – VI

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

(*Analytic Statics*)

[*Marks* : 36]

1. Answer any *one* : 12 × 1

   (*a*) Define virtual work. State and prove the principle of virtual work. Mention (with reasons) the forces which may be omitted in case of virtual work. 1 + 2 + 6 + 3

(*b*) (*i*) Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ act along the sides of the triangle formed by the lines $x + y = 1, y - x = 1$ and $y = 2$. Find the equation of the line of action of their resultant. 5

(*ii*) If a system of coplaner forces acting at different points of a body has a single resultant and if each force be turned into the plane of the forces about its point of application through the same angle, then show that their resultant will pass through a fixed point. 7

2. Answer any *two* questions : 8×2

(*a*) Show that the centroid of the arc of the helix $x = a \cos \theta, y = a \sin \theta, z = b\theta$, lying between the points $\theta = 0$, and $\theta = \alpha$ has the coordinates

$$\bar{x} = a\frac{\sin\alpha}{\alpha}, \bar{y} = a \cdot \frac{1-\cos\alpha}{\alpha}, z = \frac{1}{2}b\alpha. \quad 8$$

(*b*) Six forces, each equal to $\vec{F}$, act along the edge of a cube, taken in order, which do not meet a given diagonal. Show that their

resultant is a couple of moment $2\sqrt{3}\cdot\bar{F}a$, where 'a' is the edge of the cube. 8

(*c*) A solid frustum of paraboloid of revolution of height h and latus rectum $4a$ rests with its vertex on the vertex of a paraboloid of revolution whose latus rectum $4b$; show that equilibrium is stable if $3ab > h\,(a + b)$. 8

(*d*) A number of wrenches of the same pitch p act along generators of the same system of hyperboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1.$$

Show that they will reduce to single resultant provided their central axis is parallel to a generator of the cone

$$\left(p+\frac{bc}{a}\right)x^2+\left(p+\frac{ca}{b}\right)y^2+\left(p-\frac{ab}{c}\right)z^2=0.$$ 8

3. Answer any *two* questions : 4×2

(*a*) The paraboloid $\frac{x^2}{a}+\frac{y^2}{b}=2z$ is placed with its axis vertical and its vertex uppermost; if μ be the coefficient of friction, then show

that a particle will rest on it at any point above its curve of intersection with the cylinder $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\mu^2$. 4

(*b*) *ABCD* is a rhombus of freely joined rods lying flat on a smooth table and *P*, *Q* are the middle points of *AB*, *AD*. Prove that if the system is held in equilibrium by light strings joining *P* to *Q* and *A* to *C*, then the tensions in these strings are in the ratio of 2 *BD* to *AC*. 4

(*c*) Define 'Astatic centre'. Find its position for a system of coplanar forces acting at different points. 4

(*d*) Deduce the conditions that a given system of non-co-planar forces should be compounded into a single force. 4

GROUP – B

(*Hydrostatics*)

[*Marks* : 27]

4. Answer any *one* : 9 × 1

(*a*) What is pressure gradient ? For a fluid in

equillibrium obtain the differential equation for curves of equipressure and equidensity. 1 + 8

(*b*) A semi-ellipse bounded by its minor axis is just immersed in a liquid, the density of which varies as the depth. If the minor axis be in the surface, find the ecentricity of the semi-ellipse in order that the focus may be the centre of pressure. 9

5. Answer any *two* questions : 6 × 2

(*a*) Explain the term supprincumbent fluid. Show how to determine the thrust due to the liquid on a curved surface wholly immersed in a heavy, homogeneous liquid of rest. 6

(*b*) Show that the pressure at a point in a fluid in equilibrium is the same in every direction. 6

(*c*) A quantity of homogenous fluid is rotating with uniform angular velocity, about an axis. Find the pressure at any point in the fluid. 6

(*d*) Prove that a circular cylinder of radius '*a*' and length '*a*/*n*' can not float upright in

stable equilibrium if its specific gravity lies between

$$\frac{1}{2}\left(1-\sqrt{1-2n^2}\right) \text{ and } \frac{1}{2}\left(1+\sqrt{1-2n^2}\right).$$

Discuss the case when $2n^2 > 1$. 5 + 1

6. Answer any *two* questions : 3 × 2

(*a*) Show that in equilibrium the free surface of a liquid at rest under gravity is horizontal. 3

(*b*) Define Metacentre. Write down the basic principle and proporitions about the stability of floating body. 3

(*c*) Find the work done in compressing a gas under adiabatic change. 3

GROUP – C

(*Rigid Dynamics*)

[*Marks* : 27]

7. Answer any *one* question : 9 × 1

(*a*) Define equimomental bodies. Prove that a triangular lamina of mass M is equimomental

with four particles, three of them each of mass $\frac{M}{12}$ placed at angular points and other placed at centre of mass of the lamina. 9

(*b*) A circular ring, of mass M and radius a, lies on a smooth horizontal plane, and an insect of mass m, resting on it starts and works round it with uniform velocity v relative to the ring. Show that the center of the ring describes a circle with angular velocity $\frac{m}{M+2m} \cdot v/a$. 9

8. Answer any *two* questions: 6 × 2

(*a*) For motion of a rigid body about a fixed axis, derive the expression for kinetic energy and moment of momentum of the body. 3 + 3

(*b*) Two rods, AB and BC, of length $2a$ and $2b$, respectively, and masses proportional to their lengths, are freely joined at B and are lying in a straight line. A blow is communicated to the end A. Show that the resultant kinetic energy to the energy is $(4a + 3b)(3a + 4b) : 12(a + b)^2$ when the systm is free and C is fixed. 6

(*c*) Show that the momental ellipsoid at a point on the rim of a hemisphere is

$$2x^2 + 7(y^2 + z^2) - \frac{15}{4}xz = \text{constant}.$$ 6

(*d*) A uniform rod OA, if length $2a$, free to run about its end O, revolves with uniform angular velocity w about the vertical OZ through O, and is inclined of a constant angle α to OZ; find the value of α. 6

9. Answer any *two* questions: 3×2

(*a*) Show that the centre of suspension and the centre of oscillation of a compound pendulum are convertible. 3

(*b*) Calculate the moment of inertia of a hollow sphere of man M and radius a about a diameter. 3

(*c*) Write down the principle of independence of the motions of translation and rotation of a rigid body. 3
