2016

MATHEMATICS

[Honours]

PAPER – IV

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[OLD SYLLABUS]

GROUP – A

(*Analytical Dynamics*)

[*Marks* : 40]

1. Answer any *one* question : 15 × 1

(*a*) (*i*) A gun is mounted on a gun carriage movable on a smooth horizontal plane and the gun is elevated at an angle α to the horizontal. A shot is fired and leaves the gun in a direction inclined at an angle θ to the horizon. If the mass of the gun and its carriage be n times that of the shot, show that

$$\tan\theta = \left(1 + \frac{1}{n}\right)\tan\alpha .$$ 7

(*ii*) A heavy uniform chain of length $2l$, hangs over a small smooth fixed pulley, the length $l + c$ being at one side and $l - c$ at other side. If the end of the shorter portion be held and then let go, show by the principal of energy, that the chain will ship off the pulley in time

$$\sqrt{\frac{l}{g}} \log_e\left[\left(l + \sqrt{\left(l^2 - c^2\right)}\right)\Big/c\right].$$ 8

(*b*) (*i*) Two equal spheres lie at rest and in contact with a smooth horizontal table, when they are struck simultaneously by a third equal sphere moving at right angles to their line of centres along the table. If the last sphere is brought to rest by the impact and all the spheres are smooth, proved that the co-efficient of restitution is 2/3 and determine the fraction of the original *K*, *E* which is lost. 7

(*ii*) For a partide moving in a central orbit under inverse square law (μ/r_2). Prove

(*I*) Angular momentum about the centre of the force is constant.

(*II*) The velocity v at any distance r is given by $v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$

(*III*) Find also the period of elliptic orbit. 3 + 3 + 2

2. Answer any *two* questions : 8 × 2

(*a*) A particle of mass m is at rest and being to move under the action of a constant force F in a fixed direction. If encounters the resistance of a stream of fine dust moving in the opposite direction with velocity V, which deposits matter on it at a constant rate l. Show that its mass will be M when it has travelled a distance

$$k/l^2\left[m-M\left\{1+\log\frac{m}{M}\right\}\right]$$

where $k=F-lv$. 8

(*b*) Find the velocity and acceleration components of a particle moving on a plane with respect to a pair of rectangular axes in the plane and rotating with constant angular velocity. 8

(*c*) A ball of mass m moving under gravity in a medium which deposits matter on the ball at a uniform rate μ. Show that the equation to the trajectory, referred to horizontal and vertical axis through a point on itself may be written in the form

$$k^2uy=kx(g+kv)+gu\left(1-e^{\frac{kx}{u}}\right)$$

where u and v are the horizontal and vertical velocities at the origin and $mk = \mu$. 8

3. Answer any *three* questions : 3×3

(*a*) A mass m when suspended from a light spring causes an extension α. If a mass M is added to m, find the periodic time of the ensuring oscillation together with the amplitude of the oscillation. 3

(*b*) State the modification of Keplar's third law. 3

(*c*) A small satellite revolves round a planet of mean density 10 gm/cc, the radius of the orbit being slightly greater than the radius of the planet. Calculate the time of revolution of the satellite $G = 6{\cdot}6 \times 10^{-8}$ c.g.s units. 3

(*d*) A particle describes the equiangular spiral $r = ae^{\theta}$ in such a manner that the radial acceleration is zero. Prove that the speed and the magnitude of acceleration are each proportional to r. 3

GROUP – B

(*Linear Programming and Game Theory*)

[*Marks* : 36]

4. Answer any *one* question : 15 × 1

(*a*) (*i*) Reduce the feasible solution $x_1 = 2$, $x_2 = 1, x_3 = 1$ of the system of equations $x_1 + 4x_2 - x_3 = 5, 2x_1 + 3x_2 + x_3 = 8$ to a basic feasible solutions. 6

(*ii*) Write down the standard form of an LPP and its characteristics. 2

(*iii*) An automobile dealer is faced with the problem of determining the minimum cost policy for supplying dealers with the desired number of automobiles. The relevant data are given below. Show the procedure of solving this problem and obtain the minimum total cost of transportation. 7

The cost unit is in 100 rupees.

Dealers

		1	2	3	4	5	Plant Capacity
	A	1·2	1·7	1·6	1·8	2·4	300
Plant	B	1·8	1·5	2·2	1·2	1·6	400
	C	1·5	1·4	1·2	1.5	1·0	100
Requirement		100	50	300	150	200	

(*b*) (*i*) Use simplex method to solve the following : 8

Minimize $Z = 4x_1 + x_2$

subject to $3x_1 + x_2 = 3$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0.$$

(*ii*) A company have five jobs to be done. The matrix given below shows the return in rupees on assigning the jobs to the machines. Assign five jobs to the machines to maximize the expected return. 7

Machine

		M_1	M_2	M_3	M_4	M_5
	$J1$	32	38	40	28	40
	$J2$	40	24	28	21	36
Jobs	$J3$	41	27	33	30	37
	$J4$	22	38	41	36	36
	$J5$	29	33	40	35	39

5. Answer any *two* questions : 8×2

(*a*) (*i*) Define a basic feasible solution for a linear programming problem :

Max $Z = cx$

subject to $Ax = b, x \geq 0$. 2

(*ii*) Prove that if for a basic feasible solution X_B of the above LPP, $z_j - c_j \geq 0$ for every column a_j of A then X_B is an optimal solution. 6

(*b*) Find the optimal solution of the following L.P.P by solving its dual : 8

Minimize $Z = 4x_1 + 3x_2 + 6x_3$

subject to $x_1 + x_3 \geq 2$

$x_2 + x_3 \geq 5$

$x_1, x_2, x_3 \geq 0$

(*c*) Solve the following 4 × 2 game graphically

$$\text{Player } A \quad \overset{\text{Player } B}{\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}}$$

8

6. Answer any *one* question : 3 × 1

(*a*) Prove that the set of all feasible solutions of a linear programming problem is a convex set. 3

(*b*) Define hyper plane in R^n and then supporting hyperplane to a convex set S in R^n. 1 + 2

7. Answer any *one* question : 2 × 1

(*a*) Examine whether the set

$$X = \{(x, y) : x^2 + y^2 \leq 4\}$$

is convex or not. 2

(*b*) What do you mean by a degenerate solution ? Give an example. 2

GROUP – C

(*Tensor Calculus*)

[*Marks* : 14]

8. Answer any *one* question : 8 × 1

(*a*) (*i*) If g^{ij} is the metric tensor in a Riemannian space show that its reciprocal g^{ij} is a symmetric contravariant tensor. 4

(*ii*) If A_j^i be a mixed tensor of order 2, then prove that contracted tensor A_j^i is an invariant. 4

(*b*) (*i*) If A^{ij} is a skew symmetric tensor, show that

$$A^{ij},i = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^i}\left(\sqrt{g}A^{ij}\right) \qquad 4$$

(*ii*) Prove that

$$R_{hijk,l} + R_{hikl,j} + R_{hilj,k} = 0. \qquad 4$$

9. Answer any *two* questions : 3 × 2

(*a*) If A_i be a convariant vector, determine whether $\frac{\partial A_i}{\partial x^j}$ is a tensor or not. 3

(*b*) If the curl of a covariant vector is zero, show that the vector is a gradient. 3

(*c*) If $g^{ik}R_{kj} = R^i_j$ and $g^{ij}R_{ij} = R$ show that

$$R^i_{j,i} = \frac{1}{2}\frac{\partial R}{\partial x^i} \qquad 3$$
