2016

MATHEMATICS

[Honours]

PAPER – III

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[OLD SYLLABUS]

GROUP – A

(*Vector Analysis*)

[*Marks* : 27]

1. Answer any *one* question : 8 × 1

(*a*) (*i*) Show that the equation of the plane which contains the origin and the line of intersections of the planes $\vec{r}\cdot\vec{\alpha}=m$ and $\vec{r}\cdot\vec{\beta}=m$ is $\vec{r}\cdot(n\vec{\alpha}-m\vec{\beta})=0$. 4

(*ii*) Find the scalar area of the square, of which

$$\vec{r}=a_1\hat{i}+b_1\hat{j}$$

is a semi diagonal. 4

(*b*) (*i*) Show that the points $\vec{i}-\vec{j}+3\vec{k}$ and $3\vec{i}+3\vec{j}+3\vec{k}$ are equidistant from the plane $\vec{r}\cdot(5\vec{i}+2\vec{j}-7\vec{k})-9=0$ where $\vec{r}$ is the position under of any point on the plane. 4

(*ii*) Prove that div curl $\vec{A}=0$. 4

2. Answer any *four* questions : 4 × 4

(*a*) If $\vec{a},\vec{b},\vec{c}$ be three non-coplanar vectors and $\vec{a}',\vec{b}',\vec{c}'$ constitute the reciprocal system of vectors, then prove that any vector $\vec{r}$ can be expressed as

$$\vec{r}=(\vec{r}\cdot\vec{a}')\,\vec{a}+(\vec{r}\cdot\vec{b}')\,\vec{b}+(\vec{r}\cdot\vec{c}')\,\vec{c}$$ 4

(*b*) *Verify Stokes theorem for*

$$\vec{F} = (x^2 + y^2)\hat{i} - 2xy\,\hat{j}$$

taken round the rectangle bounded by $y = 0,\ x = a,\ y = b,\ x = 0.$ 4

(*c*) Verify Green's theorem in the plane for

$$\int_C \left\{\left(2xy - x^2\right)dx + \left(x^2 + y^2\right)dy\right\}$$

where C is the boundary of the region enclosed by $y = x^2$ and $y^2 = x$ described in the positive sense. 4

(*d*) Show that the vector function $\vec{F}$ defined by $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\,\hat{k}$ is irrotational and find a function ϕ such that $\vec{F} = \vec{\nabla}\phi$. 4

(*e*) In what direction from the point $(1, 1, -1)$, directional derivative of $f = x^2 - zy^2 + 4z^2$ is maximum ? Also find the value of this maximum directional derivative. 4

(*f*) By vector method prove that the medians of a triangle are concurrent. 4

3. Answer any *one* question : 3 × 1

(*a*) If $\vec{a}$ and $\vec{b}$ are irrotational, prove that $\vec{a} \times \vec{b}$ is solenoidal. 3

(*b*) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $\vec{i} + \vec{j} + 3\vec{k}$. 3

GROUP – B

(*Analytical Geometry*)

[*Marks* : 45]

(*Analytical Geometry of two Dimensions*)

[*Marks* : 18]

4. Answer any *two* questions : 8 × 2

(*a*) Show that the equation of the circle, which passes through the focus of the parabola

$$\frac{2a}{r} = 1 + \cos\theta$$

and touches it at the point $\theta = \alpha$ is given by

$$r\cos^3 \alpha/2 = a\cos\left(\theta - \frac{3\alpha}{2}\right)$$ 8

(*b*) Find the equations of the diagonals of the parallelogram formed by the lines

$$ax + by + c = 0, \ ax + by + d = 0,$$
$$a'x + b'y + c' = 0 \text{ and } a'x + b'y + d' = 0.$$

Show that the parallelogram will be rhombus if $(a^2 + b^2)(c' - d')^2 = (a'^2 + b'^2)(c - d)^2$. 8

(*c*) If the three normals from a point to the parabola $y^2 = 4ax$ cut the axis in points, whose distances from the vertex are in A.P., then show that the point lies on the curve $27ay^2 = 2(x - 2a)^3$. 8

5. Answer any *one* question : 2 × 1

(*a*) What is the concept of pole and polar ? 2

(*b*) Find the condition that line $lx + my + n = 0$ may be normal to the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$ 2

(*Analytical Geometry of three Dimensions*)

[*Marks* : 27]

6. Answer any *one* question : 15 × 1

(*a*) (*i*) A line moves so as to meet the lines

$$\frac{x}{\cos\alpha} = \frac{y}{\pm\sin\alpha} = \frac{z \mp c}{0}$$

in A and B, and passes through the curve $yz = k^2, x = 0$. Prove that the locus of the middle point is a third degree curve, two of whose asymptotes are parallel to the given lines. 7

(*ii*) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y + z = 0$, $z + x = 0$,

$x+y=0$, $x+y+z=0$ is $\frac{2a}{\sqrt{6}}$ and that the three lines of shortest distance intersect at the point $x=y=z=-a$. 8

(*b*) (*i*) Show that the cones $ayz+bzx+cxy=0$ and $\sqrt{ax}+\sqrt{by}+\sqrt{cz}=0$ are reciprocal, the former contains the co-ordinate axes while the latter touches the co-ordinate planes. 7

(*ii*) A variable plane is at a constant distance d from the origin and meets the co-ordinate axes in A, B, C. Show that the locus of the centroid of the tetrahedron $OABC$ is

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{d^2}.$$ 8

7. Answer any *one* question : 8 × 1

(*a*) Find the equations of the generators of the hyperboloid

$$\frac{x^2}{25}+\frac{y^2}{16}-\frac{z^2}{4}=1$$

which are parallel to the plane $8x + 10y + 20z - 11 = 0$. 8

(*b*) The vertex of a cone is (a, b, c) and yz-plane cuts it in the curve $f(y, z) = 0, x = 0$. Show that zx-plane cuts it in the curve

$$y=0, f\left(\frac{bx}{x-a}, \frac{cx-az}{x-a}\right)=0.$$ 8

8. Answer any *one* question : 4 × 1

(*a*) Find the distance of the point $(3, -4, 5)$ from the plane $2x + 5y - 6z = 16$ measured parallel to a line with direction ratios $(2, 1, -2)$. 4

(*b*) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 25, x + 2y - z + 2 = 0$ and the point $(1, 1, 1)$. 4

GROUP – C

(*Astronomy*)

[*Marks* : 18]

9. Answer any *one* question : 15 × 1

(*a*) (*i*) If two stars of known right ascensious α_1, α_2 and declinations δ_1, δ_2 rise at the same moment at a place of latitude ϕ, then show that

$$\sin^2(\alpha_1 - \alpha_2)\cot^2\phi = \tan^2\delta_1 + \tan^2\delta_2 - 2\tan\delta_1\tan\delta_2\sin(\alpha_1 - \alpha_2).$$ 8

(*ii*) Show that the duration of twilight at a place on equator is

$$\frac{12}{\pi}\sin^{-1}(\sin 18° \sec\delta)\,\text{hrs.}$$ 7

(*b*) (*i*) Find the effect of annual aberration on the latitude and longitude of a star. 8

(*ii*) If h and h' are the hour angles of a star of declination δ (north) on the (west) prime vertical and at setting respectively, for a place in north latitude, show that

$$\cos h . \cos h' + \tan^2 \delta = 0. \qquad 7$$

10. Answer any *one* question : 3 × 1

(*a*) Determine the constant of annual parallex. 3

(*b*) State Kepler's laws of planetary motion. 3