2015

ELECTRONICS

[Honours]

PAPER – I

Full Marks : 100

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[NEW SYLLABUS]

GROUP – A

Answer any **two** questions : 15 × 2

1. (*a*) Prove that 4

$$\vec{\nabla}\times\left(\vec{A}\times\vec{B}\right)=\left(\vec{B}\cdot\vec{\nabla}\right)\vec{A}-\left(\vec{A}\cdot\vec{\nabla}\right)\vec{B}+\vec{A}\left(\vec{\nabla}\cdot\vec{B}\right)-\vec{B}\left(\vec{\nabla}\cdot\vec{A}\right)$$

(*b*) Solve the differential equation : 4

$$\frac{d^2y}{dx^2} + a^2y = \sec ax$$

(*c*) Find the Laplace transform of $\frac{\sin t}{t}$. 3

(*d*) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$. 4

2. (*a*) **Explain the principle of virtual work. What is the distinction between actual and virtual work ?** 4

(*b*) **How did D' Alembert arrive at his equation using the above principle ?** 4

(*c*) **Derive Lagrange's equations for a holonomic conservative system from D' Alembert's principle.** 7

3. (*a*) The electric potential in a certain region of space is given by

$$\phi(r) = \frac{q}{4\pi \epsilon_0} \frac{e^{-r/\lambda}}{r}$$

where λ is a constant and $\vec{r}$ is the position vector. Find the corresponding electric field $E(r)$ and charge density $\rho(r)$. 8

(*b*) Suppose a point charge q is placed on the axis of a circular area of radius 'a' at a distance x from its centre. Calculate the flux of the electric field due to q through the circular area. 7

GROUP – B

Answer any **five** questions : 8×5

4. (*a*) If $w = u(x, y) + iv(x, y)$ represents the complex potential for an electric field and

$$v = x^2 - y^2 + \frac{x}{x^2 + y^2}$$

determine the function u. 4

(*b*) Use divergence theorem to evaluate

$$\iint_S \vec{A} \cdot d\vec{S}$$

where $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and S is the sphere $x^2 + y^2 + z^2 = a^2$. 4

5. (*a*) Show that the transformation

$$P = \frac{1}{2}(p^2 + q^2), \quad Q = \tan^{-1}\frac{q}{p}$$

is canonical. 4

(*b*) Prove that the shortest distance between two points in a plane is a straight line. 4

6. (*a*) prove that in Newton's ring

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}$$

for air film, other symbols have usual meaning. 4

(*b*) In a Newton's ring experiment, air in the interspace is replaced by water ($n = 1{\cdot}33$). In what proportion would the radius of the bark rings change ? 4

7. What is the function of an eyepiece ? Why does an eyepiece consist of two lenses instead of one ? Find the focal points and the principal points of a Huygen's eyepiece. 8

8. State and prove Rodrigue's formula for Legendre's polynomial. 8

9. $f(x) = x + x^2$ for $-\pi < x < \pi$. Find the Fourier expression of $f(x)$. Deduce that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\cdots$$ 8

10. A particle of mass m is moving in a plane under an inverse square law attractive force. Set up the Lagrangian and hence obtain the equation describing its motion. 8

11. (*a*) If $\vec{A} = 2y\hat{i} - 3z\hat{j} + 2x\hat{k}$.

Find A_r, A_θ, A_ϕ. 4

(*b*) Prove that

$$\iiint_v \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right) dv = \iint_S \left(\phi\vec{\nabla}\psi - \psi\vec{\nabla}\phi\right)\cdot\vec{dS}$$

where v is the volume bounded by the surface S and ϕ, ψ are scalar fields. 4

GROUP – C

Answer any **five** questions : 4×5

12. Express $J_{3/2}(x)$ and $J_{-3/2}(x)$ in terms of sine and cosine functions. $2 + 2$

13. If $f(z)$ is an analytic function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|\mathrm{Re}\, f(z)|^2 = 2|f'(z)|^2. \qquad 4$$

14. Establish the Hamiltonian of a simple pendulum and equations of motion. 4

15. Prove that force on a dipole of moment $\vec{m}$ is

$$\vec{F} = \left(\vec{m}\cdot\vec{\nabla}\right)\vec{B} \qquad 4$$

16. Find current distribution $\vec{J}$ if $\vec{A}(r,\theta,z) = \frac{k}{r^2}\hat{z}$ where k is a constant. 4

17. Explain rectilinear propagation of light according to wave theory of light. 4

18. *Show* that intensity distribution in a single slit

$$I = I_0 \frac{\sin^2 \beta}{\beta^2} \text{ where } \beta = \frac{\pi a \sin \theta}{\lambda}.$$ 4

19. Find the value of ∇^2 in spherical polar co-ordinate. 4

[*Internal Assessment* : 10 Marks]