2017

MCA

1st Semester Examination

PROBABILITY & STATISTICS

PAPER—MCA-105

Full Marks : 100

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

Answer any *five* questions : 5×4

1. The expectation of a random variable X is 50. Find the expectation of $3x + 4$. 4

2. A discrete sample space consists of only four sample points with associated probabilities.

x, $x + 1$, $2x$, $2x - 1$. Find x. 4

3. Under what condition is the following equality true ?

$P(A \cup B) = P(A) + Q(B)$.

If that condition is not true, then what will be the expression for $P(A \cup B)$? 4

4. On an average, an event X occurs two times a day. Assuming that the distribution is Poisson, find the probability that X occurs three times a day. 4

5. Find $P(2 < x < 6)$, Where the random variable x has probability density function $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$, $0 < x < \infty$

$= 0$, else where.

6. What is the probability that a four digit number has none of its digits repeated ? (A four digit number does not have 0 for the first three digits). 4

7. A coin is tossed and a six-sided die is rolled. Find the probability of getting a head on the coin and six on the die, assuming that both the coin and the die are fair. 4

Group—B

Answer any *two* questions : 2×15

8. (a) If $A_1, A_2, \ldots, A_n$ be n events conneted to a random experiment E, then

$$P(A_1 + A_2 + \ldots + A_n) \leq P(A_1) + P(A_2) + \ldots + P(A_n).$$ 7

(b) From the numbers 1, 2, 3, ... , (2n + 1), three are chosen at random. Prove that the probability that these numbers are in A.P is $\frac{3n}{4n^2 - 1}$. 8

9. (a) The probability that shooter 1 hits the target is $\frac{3}{7}$, and the probability that shooter 2 hits the target is $\frac{2}{5}$. Find the probability that at least one of them hits the target when both of them try. 5

(b) A coin is tossed two times. Find the probability that the toss results in one head and one tail. Assume the coin to be fair. 4

(c) A fair coin is tossed three times. Find the probability of getting two tails and one head. 6

10. (a) The probability density function of continuous distribution is given by

$f(x) = a(x - x^2)$, $0 \le x \le 1$, elsewhere.

$= 0$

Find the mean and variance, where a is a constant. 8

(b) The spectrum of a random variable X consists of the points 1, 2, ... , n and $P(X = i)$ is proportional to $\frac{1}{i(i+1)}$.

Compute $P(3 < x \le n)$ and $P(x > 5)$. 4+3

Group—C

Answer any *one* question : 1×20

11. (a) A *population* consists of five numbers :

2, 3, 6, 8, 11.

Consider all samples of size two. Find :

(i) the population mean.

(ii) the population variance.

(iii) The mean of the means of all possible samples.

2+3+5

(b) Compute 3 years moving averages for the following data :

Years	2005	2006	2007	2008	2009	2010
Values	11	13	16	22	4	66

5

(c) The arithmetic mean of 2, 6, x, 5 and 7 is 4. Find :

(i) x ;

(ii) the median ;

(iii) the variance. 1+1+3

12. (a) The mean of a set of values $x_1, x_2, x_3, \ldots, x_n$ is $\bar{x}$ and the variance is v. If all the values are increased by a value k, find the new mean and variance. 7

(b) If the above values are multiplied by k instead of being added, what will be the new mean and variance ? 7

(c) Under which condition will the arithmetic and geometric means of two given numbers be the same ?

(The geometric mean of two numbers a and b is $\sqrt{ab}$).

4

(d) Give three examples of measures of central tendency. Also, give one example of measures of dispersion. 2

[Internal Assessment : 30 Marks.]