2017

MCA

1st Semester Examination

DISCRETE MATHEMATICS

PAPER—MCA-102

Full Marks : 100

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer any *five* questions.

1. (a) Determine the power set ? P(A) of A = {a, b, c}. 4

(b) Find the first eight terms of the following sequence :

$a_1 = 2$, $a_2 = 4$ and $a_n = a_n - 1 + a_n - 2$ for $n > 2$. Here, a_n is the n^{th} term of the sequence. 3

(c) Find he output sequence Y for a bitwise AND gate with inputs A, B and C, where :

A = 11111000, B = 10010101, C = 00011110. 3

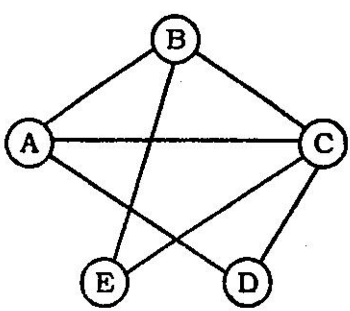
(d) Let $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Find $g[f(x)]$. 4

2. (a) Find the following matrix products : 2×2

(i) $[3 \; -2 \; 5] \begin{bmatrix} 6 \\ 1 \\ -4 \end{bmatrix}$

(ii) $[2 \; -1 \; 7 \; 4] \begin{bmatrix} 5 \\ -3 \\ -6 \\ 9 \end{bmatrix}$

(b) Consider the following graph G :



(i) Find the sets of vertices V(G) and edges E(G) 2

(ii) Find the degree of each vertex. 2½

(iii) Find the sum of the degrees of the vertices. What can you conclude on the relationship between this sum and the number of verties in this graph ? 2½

(c) Let A = {1, 2, 3, 4, 5} and B = {4, 5, 6, 7}

Find :

(i) $A \cup B$

(ii) $A \cap B$

(iii) B – A. 3

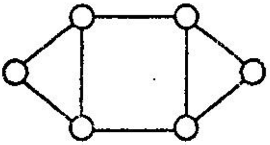
3. (a) Define the following terms with examples.

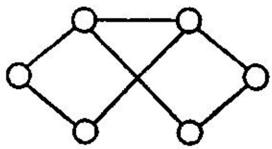
(i) Planer graph, (ii) Path (iii) Regular graph

(iv) Degree of a graph.

(b) Answer for each of these graphs.

Is it planner ? Is it bipartite ?

(i) 

(ii)  1

(c) What is adjancy matrix ? What is the best way to check for isomorphism of two graphs ?

4. (a) Express the following Boolean expressions in their complete sum of products (SOP) form :

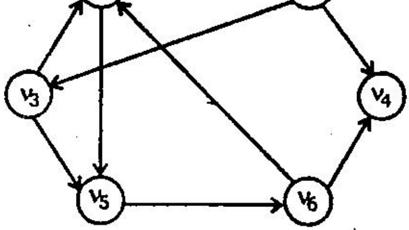
(i) $E(x, y, z) = y\left(\overline{x + yz}\right)$.

(ii) $E(x, y) = x(xy + \overline{y} + \overline{x}y)$ 2×4

(b) Find the inverse of the following metrix , if it exists. If it does not exist, state with reasons why. 6

$$\begin{bmatrix} 1 & -2 & 2 \\ 2 & -3 & 6 \\ 1 & 1 & 7 \end{bmatrix}$$

5. (a) Let G be the following directed graph.



(i) Find two simple paths from v_1 to v_6.

(ii) Find all cycles in G which include v_3.

(iii) Find the Successor list for each vertex of G.

4+3+3

(b) Using mathematical induction, prove that :

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}.$$ 4

6. (a) Let X = {1, 2, 3, 4}. With valid reasons, state whether the following relation is a function :

f = {(2, 3), (1, 4), (2, 1), (3, 2), (4, 4)} 3

(b) Let $a_n = 2^n + 5.3^n$ for n = 0, 1, 2, ...

(i) Find a_i for i = 0, 1, 2, 3, 4.

(ii) Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for all $n \geq 2,\ n \in z$.

5+6

7. (a) Find the truth table for the following boolean expression :

$xy\bar{z} + \bar{x}yz$. 5

(b) Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Verify whether det (AB) = det (BA), where det(A) means the determinant of matrix A. 4

(c) Let a be a directed graph with vertex set V(G) = {a, b, c, d, e}. The successor lists of the vertices are tabulated as follows :

Vertex	Successor list
a	b, c
b	ϕ
c	d, e
d	a, b, e
e	ϕ

Sketch (draw) the graph. 5

[Internal Assessment : 30 Marks]
