2017

M.Sc. 2nd Semester Examination

PHYSICS

PAPER—PHS-201

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer-scripts for Group-A & Group-B

Group—A

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 5×2

(a) For any vector $\vec{A}$, show that

$$\left[\vec{\sigma}, \vec{A}.\vec{\sigma}\right] = 2i\,\vec{A}\times\vec{\sigma}\,.$$

(b) If the eigenvalues of J^2 and J_z are given by $J^2 |\lambda m\rangle = \lambda |\lambda m\rangle$ and $J_z |\lambda m\rangle = m |\lambda m\rangle$, show that $\lambda \geq m^2$.

(c) The state of the hydrogen atom is 2p state. Find the energy levels of the spin-orbit interaction Hamiltonian $\lambda \vec{L}.\vec{S}$, where λ is a constant.

(d) Prove that the time reversal operator operating on any number changes it into its complex conjugate.

(e) Prove that $\text{tr.}(\gamma_\mu \gamma_\nu) = 4g_{\mu\nu}$.

(f) If $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$.

Calculate $\gamma_\mu(1-\gamma_5)(\not{p}-m)\gamma^\mu$.

(g) Show that

$$\left[\hat{H}_D, J^2\right] = 0$$

where $\hat{H}_D$ is the Dirac Hamiltonian and $\vec{J} = \vec{L} + \vec{S}$.

(h) The spin operator in the rest frame for a Dirac particle is defined by $\vec{S} = \frac{1}{2}\vec{\Sigma}$

where $\vec{\Sigma} = \frac{1}{2}\gamma \times \gamma$

Prove that $\Sigma = \gamma_5 \gamma_0 \gamma$.

2. (a) Prove that

$$j_+ |j, m\rangle = \left[j(j+1) - m(m+1)\right]^{1/2} \hbar \, |j, m+1\rangle .$$

(b) If $Y_{l,m}(\theta, \phi)$ form a complete set of orthonormal functions of (θ, ϕ).

Prove that $\sum_{l} \sum_{m=-l}^{l} |Y_{l,m}\rangle\langle Y_{l,m}| = \hat{1}$.

(c) Obtain the hyperfine splitting in the ground state of the hydrogen atom to first order in perturbation theory,

$H' = A\, \vec{S}_p . \vec{S}_e \, \delta^3\left(\vec{r}\right)$

A being constant.

Where S_p and S_e denote the spins of the proton and electron respectively ; also $\vec{F} = \vec{S}_p + \vec{S}_e$

$$\left(\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}\right).$$ 4+2+4

3. (a) If $\hat{\pi} = \hat{p} - \frac{e\vec{A}}{c}$

prove that $(\hat{\sigma}.\hat{\pi})(\hat{\sigma}.\hat{\pi}) = \left(\hat{p} - \frac{e\hat{A}}{c}\right)^2 - \frac{e\hbar}{c}\hat{\sigma}.\vec{B}$.

(b) If $\hat{K}\hbar = \beta\left(\hat{\sigma}'.\vec{L} + \hbar\right)$ where $\hat{\sigma}' = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$.

Find the eigenvalues of the operator $\hat{K}$.

(c) If $\vec{S} = \frac{1}{2}\vec{\Sigma}$ where $\vec{S}$ is the spin operator in the rest frame of a Dirac particle, then show that $S^2 = -\frac{3}{4}$. 4+4+2

Group—B

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 2×5

(a) If $L^{-1}\{F(s)\} = f(t)$

then prove that $tf(t) = L^{-1}\left\{\frac{-d}{ds}[F(s)]\right\}$.

(b) Solve

$$(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = (ly - mx).$$

(c) If a group is defined by

$$a * b = a + b - 1$$

Find the inverse of the group.

(d) Let H be a subgroup of a group G. If $x^2 \in H$ for all $x \in$ then prove that H is a normal subgroup of G.

(e) Find the direct sum $A \oplus B \oplus C$

where $A = [a]$, $B = \begin{bmatrix} b & c \\ d & e \end{bmatrix}$ and $C = \begin{bmatrix} f & g & h \\ i & j & k \\ l & m & n \end{bmatrix}$

and show that the tr. of the direct sum = tr. A + tr. B + tr. C.

(f) Show that $y(x) = 2 - x$ is a solution of the integral equation.

(g) State and prove Legrange's theorem in group.

(h) Find $L[2^t]$ when the Kernel is e^{-pt}.

2. (a) Show that

$$L\left\{\int_0^\infty \frac{\sin(xt)}{\sqrt{x}}\, dx\right\} = \frac{\pi}{(2s)^{1/2}}.$$

(b) Find the F.T. of $f(x) = \begin{cases} 1-|x|, & |x|<1 \\ 0, & |x|>1 \end{cases}$

and hence show that $\int_0^\infty \frac{\sin^4 t}{t^4} = \frac{\pi}{3}$. 5+5

3. (a) Solve the integral equation

$$y(x) = \cos x + \int_0^\pi 2\sin(x-t)\, y(t)\, dt \cdot$$

(b)

T_d	E	$8C_3$	$3C_2$	$6\sigma_d$	$6S_4$
A_1	1	1	1	1	1
A_2	1	1	1	−1	−1
E	2	−1	2	0	0
T_1	3	0	−1	−1	1
T_2	3	0	−1	1	−1

The character table for CH_4 molecule is given above. If the mumber of unchanged basis members under the operation E, C_3, C_2, σ_d, S_4 are 4, 1, 0, 2, 0 respectively. Prove that in CH_4 molecule four orbitals spans $A_1 \oplus T_2$.

5+5

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