2017

M.Sc. 4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-401

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*Functional Analysis*)

Answer Q. No. *1* and any *four* from Q. No. 2 to Q. No. 6.

Notations and Symbols have their usual meanings.

1. Answer any *four* questions : 4×2

(a) Let X, Y and Z be Banach space over the same field of scalers and G, $G_n \in BL(Z, X)$. Let $F_n(x) \to F(x)$, $x \in X$ and $G_n(z) \to G(z)$, $z \in Z$. Show that $(F_nG_n)(z) \to (FG)(z)$, $z \in Z$.

(b) Let $M : l^1 \to l^1$ be a linear map defined by

$(Mx)(i) = \sum_{i=1}^{\infty} k_{ij} x(j)$ where $x = (x(1), x(2), \ldots) \in l^1$. Suppose $\sup\left\{\sum_{i=1}^{\infty} |k_{ij}| : j \in \mathbb{N}\right\} < +\infty$. Show that M is bounded.

(c) Let H being a Hilbert space and $T \in BL(H)$ be a normal operator such that $T(x) = \alpha x$ for some $x \in H$. Then show that $T^* x = \bar{\alpha} x$.

(d) Let (X, || ||) is a Banach space and Y is a closed subspace of X then show that Y is a Banach space.

(e) In Euclidean 2-space R^2 describe geometrically the open ball centred at (0, 0) with radius 1 with respect to the norm $\| (x_1, x_2) \| = |x_1| + |x_2|$.

(f) If T is a self-adjoint operator over a Hilbert space H, show that for every natural number n, T^n is self adjoint.

2. (a) Let $\| \ \|_1$ and $\| \ \|_2$ be two norms in the NLS Euclidean 2-space R^2 defined by $\| (x_1, x_2) \|_1 = \sqrt{x_1^2 + x_2^2}$ and $\| (x_1, x_2) \|_2 = \max(|x_1|, |x_2|)$. Show that two norms are equivalent.

(b) Let X and Y $\neq \{0\}$ be normed spaces with the same scalar field where $\dim X = \infty$. Show that there is atleast one unbounded linear operator $T : X \to Y$. 4+4

3. (a) Show that dual of a normed linear space is always complete.

(b) Show that a Banach space cannot have a countably infinite basis. 4+4

4. (a) Prove that every linear operator over a finite dimensional NLS is bounded.

(b) For every $x \in X$ prove that $\|x\| = \sup_{f \in X^*} \frac{|f(x)|}{\|f\|}$, where X is a NLS and X^* be the dual space of X. 4+4

5. (a) What do you mean by a best approximation to a point x $\in$ X out of a subspace Y ot X ? (X is given as an inner product space). Let F be a subspace of an inner product space X and $x \in X$. Then show that $y \in F$ is a best approximation to x if and only if $(x - y) \perp F$. Also, show that $\text{dist}(x, F) = \langle x, x - y \rangle^{\frac{1}{2}}$.

(b) Let $E \subset X$ be closed under scalar multiplication and $x \in X$. Then $x \perp E$ if and only if $\text{dist}(x, E) = \|x\|$.

(1+3)+4

6. (a) Define a self-adjoint operator and a normal operator. Give an example to show that normal operator may not be a self-adjoint operator.

(b) Let $A \in BL(H)$ be self-adjoint where H is a Hilbert space. Then show that $\|A\| = \text{Sup}\left\{|\langle Ax, x\rangle| : \|x\| \le 1, x \in H\right\}$.

(c) Let $A \in BL(H)$, where H is a Hilbert space. Then prove that,

$$A > 0 \Leftrightarrow A \ge 0 \text{ and } \text{Ker}(A) = \{0\}$$

$$\Leftrightarrow A \ge 0 \text{ and } \overline{\text{Ran}(A)} = H,$$

where Ran(A) is the range of A. 2+3+3

[Internal Assesment : 10 Marks]