2017

M.Sc. 2nd Seme. Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-206 (Unit-I)

Full Marks : 25

Time : 1 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(General Topology)

Answer Q. No. *1* and any *two* from the rest.

1. Answer any *two* questions :   2×2

   (a) Define a order topology on an ordered set X.

   (b) Define a quotient topology with an example.

   (c) Using the definition of compactness of a topological space, show that the subspace $A = \{0\} \cup \left\{ \dfrac{1}{n} : n \in \mathbb{N} \right\}$ of $\mathbb{R}$ is compact.

2. (a) Let $\mathscr{B}$ be a basis for the topology of a non-empty set X and ρ be a basis for the topology of Y. Then show that the collection $D = \{B \times C / B \in \mathscr{B} \text{ and } C \in \rho\}$ is a basis for the topology of $X \times Y$.

(b) X and Y be two topological spaces. Suppose that $A \subset X$ and $B \subset Y$. Show that in the topological space $X \times Y$, $\overline{A \times B} = \overline{A} \times \overline{B}$.   4+4

3. (a) Let Y be an ordered set having order topology. Let f, g : $X \rightarrow Y$ be two continuous functions. Then show that the set $\{x \mid f(x) \leq g(x)\}$ is closed in X.

(b) Consider the product and box topologies on $\mathbb{R}^w$. Find the topologies for which function from $\mathbb{R}$ to R^w defined by $f(t) = \left(t, \dfrac{1}{2}t, \dfrac{1}{3}t, ...\right)$ is continuous ?   4+4.

4. (a) Show that the box topology in $\mathbb{R}^w$ is not connected.

(b) Show that a subspace of a regular topological space is regular.

(c) State the following :

(i) Urysohn Lemma   (ii) Tietze extension theorem,
(iii) Tychnoff theorem.   2+2+(1+2+1)

[Internal Assessment — 5]

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