2017

M.Sc. 2nd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-205

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*General Theory of Continuum Mechanics*)

Answer Q. No. *1* and any *four* from the rest.

1. Answer any *two* questions : 4×2

(a) The state of stress throughout a continuum is given with respect to the cartesian axes $ox_1x_2x_3$ by

$$(T_{ij}) = \begin{pmatrix} 3x_1x_2 & 5x_2^2 & 0 \\ 5x_2^2 & 0 & 2x_3 \\ 0 & 2x_3 & 0 \end{pmatrix}$$. Determine the stress vector

acting at the point $P(2, 1, \sqrt{3})$ of the plane that is tangent to the cylindrical surface $x_2^2 + x_3^2 = 4$ at P.

4

(b) Show that in two dimensional irrotational fluid motion, stream function $\psi(x, y)$ and velocity potential $\phi(x, y)$ satisfy the Laplace's equation and also show that the family of curves $\phi(x, y) = constant$ and $\psi(x, y) = constant$ cut orthogonally at their point of intersection.

4

(c) Prove that the pressure at a point in a perfect fluid has the same magnitude in every direction.

4

2. Show that the velocity potential given by

$\frac{1}{2}\log\left[\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right]$ gives a possible motion of a liquid in two-dimensional motion. Also determine the form of stream lines and the curves of equal speed. 3+3+2

3. (a) What is the current function in two-dimensional incompressible fluid motion ? 2

(b) State and prove Kelvin's Minimum Energy Theorem. 6

4. (a) Derive the integral of Euler's Equation of motion when body forces are conservative, pressure is a function of density only and flow is irrotational. 5

(b) Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$ is a possible form of boundary surface of a liquid motion. 3

5. (a) What is the concept of the principal stress and the principal direction of stress ? 2

(b) The stress tensor at P is given by $(T_{ij}) = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$.

Determine the principal stresses and principal directions of stress. 6

6. (a) Explain the geometric interpretation of infinitesimal strain tensors. 6

(b) Find the relation between α and β such that the small deformation defined by $u_1 = \alpha x_1 + 3x_2$, $u_2 = x_1 - \beta x_2$ and $u_3 = 3x_3$ is isochoric. 2

[Internal Assessment —10]

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