2017

M.Sc.

3rd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-302

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(Transorms and Integral Equations)

Answer *Q. No. 1* and any *three* questions from the rest.

1. Answer any *five* of the following questions : 5×2

(a) Find Laplace transform for the following functions when exist with reason :

(i) $\frac{1}{t}$, (ii) $\ln(t)$.

(b) Define convolution on Fourier transform.

(Turn Over)

(c) Define the inversion formula for Fourier cosine transform of the function $f(x)$. What happens, if $f(x)$ is continuous.

(d) What do you mean by Fredholm alternative in integral equation ?

(e) Define an integral equation with an example.

(f) Verify the final value theorem in connection with Laplace transform for the function $f(t) = t^3 e^{-t}$.

(g) Define the wavelet function and analyze the parameters involving in it.

(h) Find the Laplace transform of $f(x) = [x]$, where $[x]$ represents the greatest integer less than or equal to x.

2. (a) Reduce the boundary value problem $\frac{d^2y}{dx^2} + \lambda xy = 1$, $0 \le x \le l$, with boundary condition $y(0) = 0$, $y(l) = 1$ to an integral equation and find its Kernel. 5

(b) Show that if a function $f(x)$ defined on $(-\infty, \infty)$ and its Fourier transform $F(\zeta)$ are both real, then $f(x)$ is even. Also show that if $f(x)$ is real and its Fourier transform $F(\zeta)$ is purely imaginary, then $f(x)$ is odd. 3

(c) Find the exponential Fourier transform of

$$f(t)=\begin{cases} k, & -T \le t < 0 \\ -k, & 0 \le t < T \\ 0, & \text{otherwise} \end{cases}$$ 2

3. (a) Solve the following integral equation

$y(x) = f(x) + \lambda\int_{-1}^{1}(xt + x^2t^2)\, y(t)\, dt$, and find the eigen values. 5

(b) Evaluate $L\{J_0(t)\}$ by the help of initial value theorem. 5

4. (a) If the Fourier transform of $f(x)$ is $\frac{\alpha}{1+\alpha^2}$, α being the transform parameter, then find $f(x)$. 3

(b) If $L\{f(t)\} = F(p)$ which exists $Real\ (p) > \gamma$ and $H(t)$ is unit step function, then prove that for any α, $L\{H(t-\alpha) f(t-\alpha)\} = e^{-p\alpha}F(p)$ which exists for $\text{Real}(p) > \gamma$. 3

(c) Under certain conditions (to be specified by you), convert the following integral equation $f(x) = \int_0^x k(x,t)\, y(t)\, dt$ into the Volterra integral equation of second kind and then solve it. 4

5. (a) State and prove Parseval's identity on Fourier transform. Use generalization of Parseval's relation to show that

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{ab(a+b)}, a, b > 0. \qquad 4+2$$

(b) Find the resolvent Kernel of the following integral equation and then solve it :

$$\varphi(t) = t^2 + \int_0^x \sin(t-y)\phi(t)dt. \qquad 4$$

6. (a) Solve the following ODE by Laplace transform technique :

$y''(t) + 2y'(t) + 5y(t) = h(t)$ with initial condition $y(0) = 0$

and $y'(0) = 0$, where $h(t) = \begin{cases} 1, & 0 < 1 < \pi \\ 0, & t > \pi \end{cases}$ 5

(b) Define the continuous wavelet function and also explain the inverse wavelet transform. Write some important applications of wavelets.

[Internal Assessment—10 Marks]
