2017

M.Sc.

3rd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-301

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(Partial Differential Equations and Generalized Functions)

Answer *Q. No. 1* and any *two* questions from the rest.

1. Answer any *two* of the following questions : 2×4

(a) Prove the following :

(i) $\delta(-t) = \delta(t)$ and

(ii) $\delta(at) = \frac{1}{a}\delta(t), a > 0$

(Turn Over)

(b) Give an example of a Cauchy problem for a first order quasi-linear PDE which has infinitely many solution and Jacobian is identically equal to zero.

(c) Define domain of dependence for the wave equation.

Also, let $u(x, t)$ be a solution of the wave equation $u_{tt} - c^2u_{xx} = 0$, which is defined in the whole plane. Assume that u is constant on the line $x = 2 + ct$. Prove that, $u_t + cu_x = 0$.

2. (a) Consider the equation $u_{xx} + 4u_{xy} + u_x = 0$.

(i) Find the Canonical form of the equation

(ii) Find the general solution $u(x, y)$ and check by substituting back into the equation that the solution is indeed correct.

(b) Solve : $p + \frac{1}{2}q^2 = 1, u(0, y) = y^2, 0 \le y \le 1$

(c) Find the solution of $(D^2 + DD' - 6D'^2)u = y\cos x$ where

$D \equiv \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$. 8+4+4

3. (a) Establish the d'Alembert's *formula to solve the* Cauchy problem for the non-homogeneous wave equation. 8+6+2

(b) (i) Solve the problem

$$u_{tt} - u_{xx} = 0, 0 < x < \infty, t > 0$$

$$u(0,t) = \frac{t}{1+t}, 0 \leq t,$$

$$u(x,0) = u_t(x,0) = 0, 0 \leq x \leq \infty.$$

(ii) Find $\lim_{x\to\infty} u(cx, x)$ where $c > 0$.

4. (a) Solve the heat equation $u_t = 12u_{xx}$ in $0 < x < \pi$,t > 0 subject to the following boundary and initial conditions :

$$u_x(0,t) = u_x(\pi,t) = 0, t \geq 0$$

6+4+2+4

$$u(x,0) = 1 + \sin^3 x, 0 \leq x \leq \pi.$$

(b) Let $D \subseteq \mathbb{R}^2$ be a domain and $u : D \to \mathbb{R}$ be a continuous function which has the mean value property on *D*. Show that *u* is harmonic in *D*.

(c) Show that Dirac-delta function is the derivative of the Heaviside unit step function.

(d) Show that the Green's function for Laplace equation is symmetric.

[Internal Assessment—10 Marks]
