2017

M.Sc.

1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-103

Subject Code—21

Full Marks : 50

Time : 2 Hours

The figures in the right hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*ODE and Special Functions*)

Answer Q. No. *1* is compulsory and any *three* from the rest.

1. Answer any *five* questions : 5×2

(a) Define the fundamental matrix of the homogeneous system

$$\frac{d\vec{x}}{dt} = A(t)\vec{x},$$

(Turn Over)

where $\vec{x} = \vec{x}(t) = (x_1(t), x_2(t), \ldots x_n(t))^T$ and $A(t)$ is a $n \times n$ matrix.

(b) Using the integral formula for the hypergeometric function, show that

$$F(a,b,c;1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}.$$

(c) Examine that whether infinity is a regular singular point for Bessel's differential equation or not.

(d) Let $P_n(z)$ be the Legendre's polynomial of degree n. If

$$1 + z^5 = \sum_{n=0}^{5} C_n P_n(z)$$

Then find the value of C_5.

(e) Find all the singularities of the following differentital equation and then classify them :

$$z^2(z^2-1)^2\omega'' - z(1-z)\omega' + 2\omega = 0.$$

(f) Define Green's function for the non-homogeneous equation $Lu[x] = f(x)$, where L is Sturm-Liouville operator, subject to some boundary conditions at the end points of the interval $a \le x \le b$.

(g) What are Bessel's functions of order n ? State for what values of n the solutions are independent of Bessel's equation of order n.

(h) Define a self-adjoint differential equation with an example.

2. (a) Let $w_1(z)$ and $w_2(z)$ be two solutions of $(1-z^2)w''(z) - 2zw'(z) + (\sec z)w = 0$ with Wronskian $w(z)$. If $w_1(0) = 1$, $w'(0) = 0$ and $w\left(\frac{1}{2}\right) = \frac{1}{3}$, then find the value of $w_2'(z)$ at $z = 0$. 4

(b) Using Green's function method, find the solution of the equation

$$\frac{d^2u}{dx^2} = f(x), 0 \le x \le 1$$

subject to the boundary conditions $u(0) = u'(0)$ and $u(1) = -u'(1)$. 6

3. (a) If α and β are the roots of the equation $J_n(z) = 0$ then show that

$$\int_0^1 J_n(\alpha z) J_n(\beta z) dz = \begin{cases} 0, & \text{if } \alpha \neq \beta \\ \frac{1}{2}[J_n'(z)]^2 & \text{if } \alpha = \beta \end{cases}$$ 6

(b) Prove that $\int_{-1}^{1} P_m(z) P_n(z) dz = \frac{2}{2n+1} \delta_{mn}$, where δ_{mn} and $p_n(z)$ are the Kroneker delta and Legendre's polynomial respectively. 4

4. (a) Find the solution of Guass hypergeometric equation

$$z(1-z)\frac{d^2w}{dz^2} + [c - (a+b+1)z]\frac{dw}{dz} - abw = 0,$$ where a, b and c are constants. 6

(b) Show that $nP_n(z) = zP_n'(z) - P_{n-1}'(z)$, where $P_n(z)$ denotes the Legendre's Polynomial of degree n. 4

5. (a) How do you solve the homogeneous vector differential equation in the form $\frac{dx}{dt} = Ax$, where $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ and $A = (a_{ij})_{n\times n}$ matrix. [Assuming that the eigen values of A are all real and distinct.] 6

(b) Obtain the first five terms in the expansion of the following function f in terms of Legendre's polynomial

$$f(x) = \begin{cases} 0, & \text{if} \quad -1 < x < 0 \\ x, & \text{if} \quad 0 < x < 1 \end{cases}$$ 4

6. (a) Find the characteristics value and characteristic functions of the Sturm-Liouville problem $(x^3y')' + \lambda xy = 0\,; y(1) = 0, y(e) = 0$. 5

(b) Show that $J_n(-z) = (-1)^n J_n(z)$, where n is an integer. 3

(c) Prove that $\int_0^\infty \frac{J_n(z)}{z} dz = \frac{1}{n}, (n \neq 0)$. 2

(Internal Assessment : 10 Marks)
