2017

M.Sc.

1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-101

Subject Code—21

Full Marks : 50

Time : 2 Hours

The figures in the right hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

[Real Analysis]

Answer Q. No. *1* and any *four* from Q. No. *2* to Q. No. *7*.

1. Answer any *four* questions : 4×2

(a) Give an exmaple with justifications of a metric space which is closed and bounded but not compact.

(b) Verify whether the following subset 5 of $\mathbb{R}^2$ is connected or not, where $S = \{(x, y) \in \mathbb{R}^2 : xy = 1\}$.

(c) Give an example of a function which is of bounded variation defined on a closed and bounded interval, but not a Lipschitz function.

(d) Define measurability of a Borel set.

(e) Show that any function from a discrete metric space into a metric space is continuous.

2. (a) Let f : [a, b] → ℝ be a function of bounded variation on [a, c] and [c, b] where $c \in (a, b)$. Then show that

(i) f is a function of bounded variation on $[a, b]$, and

(ii) $V_f[a,c] + V_f[c,b] = V_f[a,b]$.

(b) Let f : [0, 1] → ℝ be defined by

$$f(x_n) = \begin{cases} \dfrac{1}{n^2}, & n = 1, 2, 3, \ldots \\ 0, & \text{elswhere,} \end{cases}$$

where $x_1, x_2, \ldots, x_n$ be an enumeration of all rationals in [0, 1]. Show that f is a function of bounded variation on [0, 1]. 5+3

3. (a) State and prove the second Mean-value theorem for Riemann-Stieltjes integral.

(b) Show that every path connected metric space is connected. 4+4

4. (a) Let (X_1, d_1) and (Y, d_2) be two metric space and let $f : (X_1, d_1) \to (Y, d_2)$ be a continuous function, where X_1 is a compact metric space. Prove that f is uniformly continuous on X_1.

(b) Evaluate : $\int_{-1}^{3} x^7 d\left(|x|^3 + |x|\right)$ 4+4

5. (a) Let $f : X \to \mathbb{R}^*$ be a non-negative measurable function such that $\int_E f du = 0$, where E is a measurable subset of X. Then show that $f = 0$ a.e. on X.

(b) Suppose $\{f_n\} : X \to [0 + \infty]$ is a sequence of non-negative functions measurable for $n = 1, 2, 3, \ldots$, satisfying $f_1 \geq f_2 \geq f_3 \geq \cdots$, such that $f_n(x) \to f(x)$ as $n \to \infty$, for every $x \in X$ and $f_1 \in L'(\mu)$. Prove that

$$\lim_{n \to \infty} \int_X f_n du = \int_X f du$$ 3+5

6. (a) If $\int_a^b f dx$ is Riemann integrable, prove that it is Lebesgue integrable and the two integrals are equal. Give an example which is Lebesgue integrable but not Riemann integrable.

(b) State Lebesgue Dominated Convergence theorem. 6+2

7. (a) Define a Cantor set. Show that it is uncountable but has measure zero.

(b) Let μ be a positive measure on a σ - algebra m. Prove that $\mu(A_n) \to \mu(A)$ as $n \to \infty$ where $A = \bigcup_{n=1}^{\infty} A_n, A_n \in m$ and $A_1 \subset A_2 \subset A_3 \subset \cdots \subset \cdots$. 3+5

(Internal Assessment : 10 Marks)

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