2017

M.Sc. 4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-405 (Unit-I)

Full Marks : 25

Time : 1 Hour

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper : *(Dynamical Meteorology-II / Operational Research Modelling-II)*

Operational Research Modeling—II

Answer Q. No. *1* and any *two* from the rest.

1. Answer any *two* questions : 2×2

(a) Find the curve x = x(t) which minimize the functional

$$J = \int_0^1 \left(\dot{x}^2 + 1\right) dt$$

x(0) = 1 and x(1) = 2.

(b) What is failure rate ? If the failure distribution Q has a density and failure rate $\lambda(t)$, show that

$$1 - Q(t) = \exp\left[-\int_0^t \lambda(t)dt\right]. \qquad 2$$

(c) Prove that the entropy function

$H(p_1, p_2, \ldots p_n)$ is continuous in $p_k \ \forall \ 0 \leq p_k \leq 1$. 2

2. (a) Suppose a system contains a primary element and a stand-by element. Let λ_p and λ_d represent the failure rates of primary element and stand-by element. Find the reliability of this system. Also, find the system reliability and MTBF when $\lambda_p = \lambda_d = \lambda$. 4

(b) An industrial process is controlled by a computer and two similar components are operated in stand-by redundancy such that if a computer fails another is instantaneously brought into use in its place. The failure rate of each computer is given by $\lambda = 0.01$ failure/hour. Compare the improvement in reliability over a single computer when one and then two computers are used in a stand-by. The operating period is 100 hours and the switch is considered to be perfect. 4

3. Define joint and conditional entropies. Prove that $H(X, Y) = H(X/Y) + H(Y) = H(Y/X) + H(X)$, where $H(X) \geq H(X/Y)$. 8

4. Answer any *one* question :

(a) Let a car be derived from a stationary position on a horizontal way to a stationary position in a garage moving a

total distance a. The available control for the driver the accelerator and the break. Find the minimum time to bring the car in the stationary position at a distance a and the optimal control to be applied on the car. 8

(b) There are 4 jobs each of which has to go through the machines M_i, i = 1, 2, ... 6 in the order M_1M_2 ... M_6. Processing times are given below :

		Machines					
		M_1	M_2	M_3	M_4	M_5	M_6
	A	20	10	9	4	12	27
Jobs	B	19	8	11	8	10	21
	C	13	7	10	7	9	17
	D	22	6	5	6	10	14

Determine a sequence of these four jobs which minimizes the total elapsed time. 8

[Internal Assesment : 05 Marks]

Dynamical Meteorology—II

Answer Q. No. *1* and any *two* from the rest.

1. Answer any *two* questions : 2×2

(a) Show that in a geostrophic wind field, an ideal front is necessarily stationary. 2

(b) Derive a relation between pressure difference at the top and bottom of a hurricane. 2

2. (a) Derive the pressure tendency below a frontal surface. 5

(b) What do you mean by turbulent transfer of momentum ? 2

(c) What is the concept of numerical weather prediction ? 2

3. (a) Show that the tangential velocity in a hurricane varies with altitude z by the following relation :

$$\left(\frac{2M_{tan}}{R} + f_c\right)\frac{\partial M_{tan}}{\partial z} = \frac{g}{T}\frac{\partial T}{\partial R}$$

where M_{tan} be the tangential velocity of the hurricane, R is the distance from the eye, T is the absolute temperature, f_c is the coriolis parameter. Hence show that the hurricane has a warm core. 6

(b) Derive the angle between two arms of 'V' made by the isobars at a front. 3

4. (a) Derive the meridional temperature gradient caused by global circulation in the atmosphere. 4

(b) How Jet stream and Rossby wave are developed in the atmosphere ? Derive the expression of the parameter β. 2+2+1

[Internal Assesment : 05 Marks]