2017

M.Sc. 4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-404 (OR/OM)

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*Non-linear Optimization / Dynamical Oceanology-II*)

MTM-404 (OR)

(Non-linear Optimization)

Answer Q. No. *1* and any *three* from the rest.

1. Answer any *five* from the following : 5×2

(a) Define Nash equilibrium solution and Nash equilibrium outcome in pure strategy for bimatrix game.

(b) What is stochastic programming problem ? Who first defined chance constrained programming technique and in which year ?

(c) State Slater's constrait qualification.

(d) Write the advantages and disadvantages of geometric programming.

(e) What is degree of difficulty ? Define various types of degree of difficulty.

(f) Define multi-objective non-linear programming problem with an example.

(g) What are the differences between Beale's method and Wolfe's method for solving quadratic programming problem.

2. (a) Solve the following quadralic programming problem using Beale's method

Maximize $z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

subject to $x_1 + 2x_2 \leq 10,$

$x_1 + x_2 \leq 9,$

$x_1, x_2 \geq 0$ 7

(b) Define the following :

(i) the (primal) minimization problem (MP)

(ii) the dual (maximization) problem (DP). 3

3. (a) State and prove Fritz-John saddle point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of Fritz-John saddle point problem. 6

(b) State Farkas' theorem. Give the geometrical interpretation of it. 4

4. (a) Describe the solution procedure of an unconstraied geometric programming problem (problem to be contidered by you). 7

(b) Prove that all strategically equivalent bimatrix games have the same Nash equilibria. 3

5. (a) Use the chance constrained programming technique to find an equivalent deterministic form of stochastic programming problem :

$$\text{Minimize } f(x) = \sum_{j=1}^{n} c_j x_j$$

$$P\left[\sum_{j=1}^{n} a_{ij} x_j \le b_i\right] \ge p_i \, , \; i = 1, 2, \ldots, m$$

and $x_j \ge 0, \; j = 1, 2, \ldots, n$

where a_{ij} is normally distributed random variable. 6

(b) Prove that if $(\bar{x}, \bar{u})$ is a solution of KTP then is a solution of MP, where $\bar{x} \in x^0$. Given x^0 is open in $\mathbb{R}^n$ and Q and g are differentiable and convex at $\bar{x}$. Hence prove that if $(\bar{x}, \bar{r}_0, \bar{r})$ is a solution of FJP and $\bar{r}_0 > 0$, then $\bar{x}$ is a solution of MP. 4

6. (a) Let Q be a numerical differentiable function on an open convex set $\Gamma \subset \mathbb{R}^n$. Prove that Q is Γ convex on Γ if and only if $Q(x^2) - Q(x^1) \ge \nabla Q(x^1)(x^2 - x^1)$ for each $x^1, x^2 \in \Gamma$. 6

(b) Define constraint qualifications and explain the cause of their introduction in the theory of non-linear programming problem. 4

[Internal Assesment : 10 Marks]

MTM-404 (OM)

(Dynamical Oceanology-II)

Answer Q. No. *1* and any *four* from the rest.

1. Answer any *four* questions out of six questions : 4×2

(a) Write the depth-averaged 2D shallow water equations and then derive the vorticity equation.

(b) Write the y-momentum equation in isobaric coordinate system and also write the physical interpretation of each terms.

(c) State the Taylor-Proudman theorem.

(d) Show the position of three cells of general circulation in the globe.

(e) Write the kinematic and dynamic boundary conditions at the surfaces of the shallow water.

(f) Calculate the Rossby radius of deformation for a deep ocean with height = 4000m and a shallow ocean with height = 100m.

2. (a) Calculate the circulation within a small fluid element with area $\delta x \delta y$.

(b) Consider an arbitrary large fluid element, and divide it into small squares. Then calculate the circulation within the area.

(c) Using this formula, find the circulation of the linear shear flow. 3+3+2

3. (a) Summarize equations for Quasi-Geostrophic approximations in terms of geostropic and ageostropic components of the velocity.

(b) Discuss the sequence at which the solution of above equations can be done. 5+3

4. (a) Write the thermodynamic equation in rectangular coordinate system and then convert it to its isobaric coordinate system. What is the physical interpretation of each term ?

(b) Finally reduce the above equation in terms of static stability parameter. 6+2

5. (a) Sketch the configuration of inviscid shallow water rotating about the z-axis with constant angular velocity $\Omega\sin\varphi$, where the symbols have their usual meaning.

(b) Stating the basic assumptions in shallow water theory, derive only the equation for the surface elevation.

2+6

6. (a) Stating the basic assumptions for Kelvin waves at a straight coast, write the governing equations.

(b) Find the solution of the above equations for the surface elevation and the velocity distribution. 3+5

7. (a) Stating the basic assumptions for linear waves with rotation, write the governing equations.

(b) Derive the Klein-Gordon equation.

(c) Assuming the necessary assumptions, find the stationary solution for the case with initial surface elevation as

$$\delta_0 = \begin{cases} \frac{h}{z} & \text{for } x > 0 \\ -\frac{h}{2} & \text{for } x < 0 \end{cases}.$$

Also sketch the solution.

2+3+3

[Internal Assesment : 10 Marks]

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