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Water Scarcity and Declining Trend in Yield in Foodgrains Production of India

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Abstract

India has achieved remarkable success in foodgrains production in the last four decades largely banking on tube-well irrigation. But the country is experiencing a declining trend in the growth rate in yield in recent years. This paper demonstrates theoretically how excess depletion and under utilization of ground water make water scarcity a serious constraint to future growth in agriculture. Using time series econometric analysis based on Indian data it establishes that there is meaningful long-run relationship between tube-well irrigation, fertilizer use and productivity growth in agriculture and it explains the declining trend in the growth rate in yield in terms of declining growth rate in tube-well irrigation.

JEL Classification : Q_{01}, Q_{15}, Q_{25}

Key Words : ground water, excess depletion, government support, natural conditions, unsustainable growth, cointegration

1. INTRODUCTION

The LDCs have achieved remarkable success in foodgrains production in the last few decades and it has been done largely banking on ground water extraction. The ground water irrigation has greatly facilitated the use of high-yielding varieties (HYV) seeds and chemical fertilisers in the cultivation of rice, wheat, maize and sorghum in the developing countries. The spectacular increase in productivity in foodgrains production can be attributed to the expansion of tube-well irrigation. The yield per hectare in foodgrains production in India has increased from 872 kilogram in 1970-1971 to 2059 kilogram in 2011-2012 and in this productivity growth, tube-well irrigation and fertilizer use have played a key role. In net irrigated area, the share of well-irrigation has increased from 12.34% to 60.86% during this period (CMIE, 2010). The favourable geo-physical conditions, higher productivity of HYV seeds and various government support measures have prompted huge private investment on tube-well irrigation in the country. As a result, the number of shallow and deep tube-wells for irrigation has increased significantly in the country. The Report on the Minor Irrigation Census (2000-01) in West Bengal, an eastern state of India that has recorded remarkable growth in paddy cultivation, reveals that 94% of the tube-well irrigation schemes in the state are privately owned. This gives an idea of the size of private investment on tube-well irrigation and extraction of ground water for cultivation. No doubt, ground water extraction has been helpful for agricultural growth but at the same time, rampant digging of tube-wells and uncontrolled extraction of ground water have weakened the resource base in many cases and shortage of water supply has become a serious constraint to future growth in agriculture

(Singh, 2000; Rao, 2002; Singh, 1992; Sidhu, 2002; Sasmal, 2012). The excess depletion of ground water has resulted in salinity and arsenic problems in water, decline in water table in the aquifer and degradation of soil fertility in many parts of the country. Punjab, a foodbed of India where the green revolution technology was most successfully implemented in 1960s, is found to be worst affected by the process of ground water extraction and intensive cultivation. Rosegrant and Sombilla (1997) have pointed out that the major threat that may come in the way of future foodgrains production will be the shortage of water supply. Given the natural conditions and productivity of modern technology, subsidy on inputs and price support for crops by the government have significantly influenced the ground water extraction and agricultural growth in the country. It is being suggested that emphasis should be given on rain water harvesting and surface water management, higher efficiency in irrigation system and development of rainfed agriculture. These policies are getting importance and some success has also been achieved in this regard. Nevertheless, ground water irrigation still remains the main driving force of growth in Indian agriculture.

This paper is concerned with the shortage of water supply and its impact on the sustainability of growth in foodgrains production. The private agents make under valuation of natural resources and disregard environmental and ecological costs of water extraction. The public support measures further aggravate the situation by encouraging excess depletion of the resource. In cases, where agriculture is dependent on ground water irrigation, if extraction of water exceeds the natural rate of recharge of the aquifer, the water stock gradually declines making agricultural growth unsustainable in the long run. The productivity growth in foodgrains production of India is showing a declining trend in the recent years and in many cases, it has been associated with declining growth rates of tube-well irrigation. It is not always the case that the declining growth rate of tube-well irrigation is the outcome of excess depletion of ground water. May be, there is sufficient ground water in the region but that is not utilized due to lack of investment and proper technology or because of natural and geo-physical reasons. This paper is trying to show that the declining trend in yield is the outcome of scarcity of water supply and consequent decline in the rate of fertilizer use. The work has been arranged as follows : In Section II, a theoretical model has been constructed to demonstrate how excess depletion or under utilization of ground water becomes a constraint to future growth in agriculture. In Section III the theoretical results have been empirically verified by time series econometric analysis based on Indian data. Section IV gives the summary.

2. Water Shortage And Unsustainable Growth In A Ground Water Based Agriculture

The Model

Let us consider an agrarian system where production is based on extraction of ground water in a decentralised framework. The production function can be specified as

$$Q = e(G)\,F(W, Z) \tag{1}$$

where Q is agricultural output, W is extraction of ground water and Z is other input, say, chemical fertilizer with $F_W > 0$, $F_{WW} < 0$, $F_Z > 0$, $F_{ZZ} < 0$. e is efficiency from public investment, G and $e > 1$, $e'(G) > 0$, $e''(G) < 0$. If public investment is high, productivity of water as well as of agriculture will be also high. The rain water helps agricultural production but it is assumed that the availability of rain water for agriculture is constant due to given rainfall and other natural conditions. The cost of ground water extraction per unit is C and it can be written as a function of stock of water (S) and extraction of water (W), natural conditions (N) and irrigation technology (T). The cost function can be written as

$$C = C\left(W, S, N, T\right) \qquad (2)$$

with $C_W \rangle\, 0,\ C_{WW} \rangle\, 0,\ C_S \langle\, 0, C_{SS} \langle\, 0, C_N \langle\, 0, C_T \langle\, 0$.

The irrigation technology is provided by the government. Improved technology and favourable natural conditions reduce cost of water. It may be assumed that agriculture and irrigation are subsidized.

The farmer's income is defined as

$$\pi = P \cdot e(G) \cdot F\,(\cdot) - C\,(\cdot) \cdot W - q \cdot Z + \eta \cdot W \qquad (3)$$

where P is the price of the crop, q is the price of Z. η is irrigation subsidy per unit of water. The extraction of water has some adverse impact on the environment and ecology. But private individual disregards the costs of such effects. The utility function of the household is

$U = f\,(\mu, E)$

where μ is consumption and E is environmental quality. The consumption depends on income and it may be assumed that the whole income is spent on consumption. Environmental quality is a public good and an individual can not influence it. Therefore, it can be dropped from the function. Therefore, utility function can be written as

$$U = f\,(\pi) \qquad (4)$$

The dynamics of water stock in the aquifer is

$$\dot{S} = -W + R(N) \qquad (5)$$

Where R is natural recharge to the aquifer at each point of time and it depends on natural and geo-physical conditions of the region, denoted by N. In a particular reason, N is given. So, R is fixed. But across regions, R may vary. Here, $R'(N) > 0$ implying that in a favourable geo-physical condition, recharge rate is higher.

The objective of the farmer is :

$$\text{Max} \int_0^{\propto} \pi \cdot e^{-\rho t} \cdot dt \qquad (6)$$

s.t. $\dot{S} = -W + R\left(N\right)$

$S\,(0) = S_0$, $S\,(T)$ free,

lim T → ∝

where ρ is the rate of discount of future utility.

It is a dynamic optimisation problem over a planning horizon [0 ∝] that can be solved by using optimal control theory as specified in Chiang (1992) and Dorfman (1969).

The current value Hamiltonian is

$$H = P \cdot e(G) \cdot F(\cdot) - C(\cdot) \cdot W - q \cdot Z + \eta \cdot W + \lambda\left(-W + R(N)\right) \qquad (7)$$

S is state variables and λ is costate variable. λ is the present value shadow price of S.

F.O.C.s for maximisation of H are :

$$\frac{\delta H}{\delta W} = P \cdot e(G) \cdot F_W - C - C_W \cdot W + \eta - \lambda = 0 \qquad (8)$$

$$\frac{\delta H}{\delta Z} = P \cdot e(G) \cdot F_z - q \ = \ 0 \tag{9}$$

$$-\frac{\delta H}{\delta S} = \dot{\lambda} = \rho\lambda + W \cdot C_S \tag{10}$$

$$\frac{\delta H}{\delta \lambda} = \dot{S} = -W + R\,(N) \tag{11}$$

The transversality conditions :

$$\lambda\,(T) \geq 0, \ \ S\,(T)\,\lambda\,(T) = 0$$

$\lim T \to \propto$

S.O.C. is satisfied by the strict concavity of *H* in *W*, *Z* and *S* jointly (See appendix – A). Now, the theorems of Steinberg and Stalford (1973) and Gale and Nikaido (1965) guarantee the globally and uniquely determined optimal values of the control variables in terms of state and costate variables and set of parameters as

$$\hat{W} = \hat{W}\,(S, \lambda, P, q, \eta, \rho)$$

$$\hat{Z} \ = \ \hat{Z}\,(S, \lambda, P, q, \eta, \rho)$$

The marginal condition in (8) determines the optimal value of ground water extraction at each of time by equating the marginal cost of W with its marginal benefit. Here, $\left(C + C_W \cdot W\right)$ is the direct cost of water extraction. λ is the current value shadow price of ground water stock and it measures the cost of not preserving the resource for future use. The marginal return from water extraction is $\left(P \cdot e(G) \cdot F_W + \eta\right)$. This return will be higher if irrigation subsidy (η) is higher, *P* is higher due to price support of the government and higher productivity of water due to higher public investment in agriculture (*G*). On the other hand, if natural conditions are favourable, the cost of water extraction will be lower. That means, *C* will be lower. If natural conditions are not favourable, irrigation technology (T) is not efficient and public investment for extraction of water is not sufficient, cost of water extraction (C) will be high. Thus water extraction is determined by all these factors. The resulting system of equations (8) – (11) will give the optimal paths for S, λ, W and Z. Since Q is linked with these variables in the system, its optimal path is also obtained from these equations. Therefore, the solution to the problem in (6) can be described by the differential equations in (10) and (11) along with the transversality conditions. Now, we are interested to see whether the solution to the optimization problem in (6) yields a sustainable growth path. For sustainability, we need $\dot{S} = 0.$ That means, the path of the control variable W will be such that water stock (S) remains unchanged.

Since the private individuals make under valuation of natural resources, not only the value of λ will be low but also the value will decline over time. Furthermore, if *P*, *e*(*G*), F_W, η are high due to government support and C is low due to the factors mentioned above, there is high possibility that *W* will exceed *R* (*N*) making $\dot{S}$ negative i.e. *W* > *R* (*N*) and $\dot{S} < 0$. Since there are government intervention and externality problems (private agents disregards external costs), market failure will be there. Thus if the depletion of ground water exceeds its efficient level, the excess depletion of water will make $\dot{S} \langle 0$. The water stock in the aquifer will decline over time and water scarcity will eventually make agricultural growth unsustainable in the long run. If extraction is W_1 and W_2 in Figure 1, there is excess depletion.

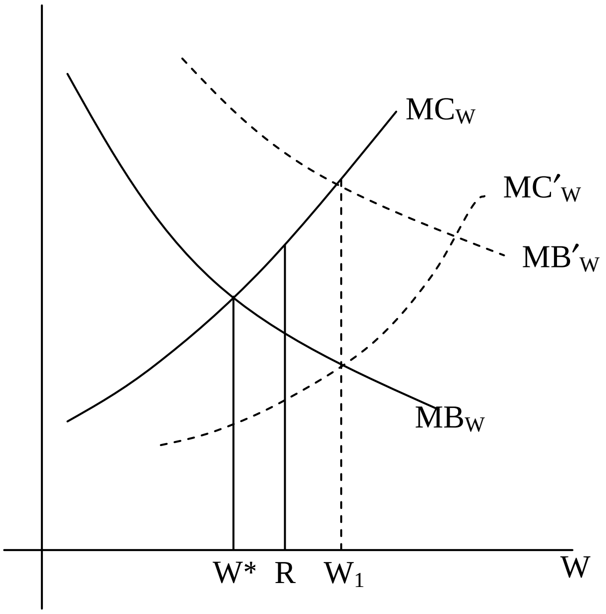

Figure : 1

Figure 1 shows that optimal water extraction (W*) is determined by marginal benefit and costs of water extraction. If marginal benefit is increased or marginal cost is reduced by government intervention, then *W* may exceed *R* with the result that $\dot{S} \langle 0$. On the other hand, marginal cost may be high and marginal benefit may be low due to lack of sufficient public investment and appropriate technology and non-favourable geo-physical conditions. In that case, sufficient water can not extracted for irrigation despite availability of water in the aquifer. Here also, scarcity of water will be a constraint to agricultural production. In Figure 1, it is $W^* < R$.

3. Econometric Results

This section presents the results of time series econometric analyses based on Indian data and examines the relationship between tube-well irrigation, fertilizer use and productivity in foodgrains production. The test of cointegration and estimation of vector error correction have been done following the techniques outlined in Enders (2004) and using the annual data on yield per hectare in foodgrains production (YIELD) and percentage share of tube-well irrigation in net irrigated area (T_WELL_IRRI) and fertilizer use (FERT) for 38 years from 1970-1971 to 2007-2008 in the Indian context. The data have been collected from Economic Survey, Ministry of Finance, Government of India and Centre for Monitoring Indian Economy (CMIE). In the Augmented Dicky Fuller (ADF) test, the variables are non-stationary at level but stationary at first difference. In a two variable vector autoregression (VAR) analysis, we can let the time path of $\{y_t\}$ be affected by current and past realizations of the $\{Z_t\}$ and let the time path of the $\{Z_t\}$ sequence be affected by current and past realizations of $\{y_t\}$ sequence. After simplification, VAR can be expressed in standard form as

(i) $y_t = a_{10} + a_{11} y_{t-1} + a_{12} Z_{t-1} + e_{1t}$

(ii) $Z_t = a_{20} + a_{21} y_{t-1} + a_{22} Z_{t-1} + e_{2t}$

A principal feature of cointegrated variables in VAR is that their time paths are influenced by the extent of any deviation from long-run equilibrium. After all, if the system is to return to the long-run equilibrium, the movement of at least some of the variables must respond to the magnitude of the disequilibrium. The variables may be non-stationary but it is possible that a linear combination of integrated variables is stationary. Such variables are said to be cointegrated.

According to the methodology outlined in Enders (2004) the error-correction representation necessitates that the two variables be cointegrated of order CI (1, 1). In *n*-variable model, the ($n \times 1$) vector $x_t = \{ x_{1t}, x_{2t}, ... x_{nt} \}'$ has an error-correction if it can be expressed as

(iii) $\Delta xt = \prod_0 + \prod \Delta x_{t-1} + \prod_1 \Delta x_{t-2} + ... + \prod_p \Delta x_{t-p} + \in_t$

Where Π_0 = an ($n \times 1$) vector of intercept terms.

Π_i = ($n \times n$) coefficient matrices with elements Π_{jk} (i)

Π = a matrix with elements Π_{jk} such that one or more of the $\Pi_{jk} \neq 0$

$\in_t$ = an ($n \times 1$) vector with elements $\in_{it}$

Let all variables in x_t be I(1). Then error-correction equation yields to

(iv) $\prod x_{t-1} = \Delta x_t - \prod_0 - \sum \pi_i \Delta x_{t-i} - \in_t$

Since each series x_{it-1} is I(1), (Π_{11}, Π_{12} ……….. Π_{1n}) must be a cointegrating vector of x_t. The results in our empirical study show that the variables are cointegrated i.e., CI (1,1). That means, there is meaningful long-run relationship between (i) yield in foodgrains production and tube-well irrigation, (ii) fertilizer use and yield and (iii) tube-well irrigation and fertilizer use (see Sasmal, 2012). The estimates of vector error correction indicate that the deviations from the long-run relationship are accounted for by the variables in lags and the residual terms.

The results of vector error correction in Table 1 show that the deviation from the long-run relationship causes movement in DYIELD and DT_WELL_IRRI. Here, e_{t-1} has significant effect on DYIELD and DT_WELL_IRRI. DYIELD (–1), DYIELD (–2) and DT_WELL_IRRI (–1) have also significant effect on DT_WELL_IRRI. Similar co-integrating relationship is found between YIELD and FERT and the deviation from the long-run relationship is accounted for by the residual term and the variables in lags in Table 2. e_{t-1} has significant effect on DYIELD and DFERT. DYIELD(–2) has significant effect on DYIELD and the effect of DT_WELL_IRRI (–1) on DYIELD and DFERT is also significant. T_WELL_IRRI and FERT are cointegrated and in error correction in Table 3, e_{t-1} and the variables in different lags explain the deviations from the long-run relationship. Thus, the empirical results establish meaningful long-run relationship between yield in foodgrains production, tube-well irrigation and fertilizer use and the variables account for short run deviations from long run equilibrium paths and relationships.

Table 1. **Cointegration between yield (YIELD) and Tube-well irrigation (T_WELL_IRRI) and vector error correction in VAR framework.**

Vector Error Correction Estimates
Sample (adjusted): 1974 2007
Included observations: 34 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
DYEILD(–1)	1.000000
DT_WELL_IRRI(–1)	–4.829026 (4.35045) [–1.11001]
C	–24.56719

Error Correction:	D(DYIELD)	D(DT_WELL_IRRI)
CointEq1	–0.936424 (0.26792) [–3.49512*]	0.028605 (0.01267) [2.25728*]
D(DYIELD(–1))	0.036036 (0.23598) [0.15271]	–0.037077 (0.01116) [–3.32184*]
D(DYIELD(–2))	0.266664 (0.18147) [1.46945]	–0.032860 (0.00858) [–3.82828*]
D(DT_WELL_IRRI(–1))	–4.377590 (3.24288) [–1.34991]	–0.594699 (0.15338) [–3.87716*]
D(DT_WELL_IRRI(–2))	–0.216026 (3.23361) [–0.06681]	–0.181298 (0.15295) [–1.18537]
C	1.369518 (4.74622) [0.28855]	0.127804 (0.22449) [0.56930]
R-squared	0.582405	0.565536
Adj. R-squared	0.507834	0.487953

F-statistic	7.810111	7.289435
Log likelihood	–157.5031	–53.76007
Akaike AIC	0.617831	3.515298
Schwarz SC	9.887189	3.784656

* denotes significant at 5% level.

Table 2. Cointegration between yield (YIELD) and fertilizer (FERT) and vector error correction in VAR framework.

Vector Error Correction Estimates
Sample (adjusted): 1974 2007
Included observations: 34 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
DYEILD(–1)	1.000000
DFERT(–1)	0.303601 (1.48661) [0.20422]
C	–28.64233

Error Correction:	D(DYIELD)	D(DFERT)
CointEq1	–1.105515 (0.24477) [–4.51654*]	–0.062523 (0.04003) [–1.56201]
D(DYIELD(–1))	0.175261 (0.21135) [0.82925]	0.082723 (0.03456) [2.39347*]
D(DYIELD(–2))	0.367993 (0.15472) [2.37840*]	0.095281 (0.02530) [3.76579*]
D(DT_WELL_IRRI(–1))	2.488385 (0.94664) [2.62864*]	–0.549069 (0.15480) [–3.54687*]
D(DT_WELL_IRRI(–2))	0.136758 (1.02540) [0.13337]	–0.238315 (0.16768) [–1.42124]

C	0.740763 (4.07616) [0.18173]	0.049893 (0.66657) [0.07485]
R-squared	0.694474	0.514901
Adj. R-squared	0.639916	0.428276
F-statistic	12.72907	5.944042
Log likelihood	−152.1910	−90.62497
Akaike AIC	9.305352	5.683822
Schwarz SC	9.574710	5.953180

* denotes significant at 5% level.

Table 3. **Cointegration between Tube-well irrigation (T_WELL_IRRI) and fertilizer use (FERT) and vector error correction in VAR framework.**

Vector Error Correction Estimates
Sample (adjusted): 1974 2007
Included observations: 34 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
DT_WELL_IRRI(−1)	1.000000
DFERT(−1)	−0.902378 (0.27076) [−3.33279]
C	1.852850

Error Correction:	D(DT_WELL_IRRI)	D(DFERT)
CointEq1	−0.371168 (0.09641) [−3.84979*]	0.763076 (0.31571) [2.41701*]
D(DT_WELL_IRRI(−1))	−0.372624 (0.17000) [−2.19195*]	−1.439108 (0.55667) [−2.58522*]
D(DT_WELL_IRRI(−2))	−0.187594 (0.15948) [−1.17627]	−0.781382 (0.52224) [−1.49622]
D(DFERT(−1))	−0.137023	−0.218730

	(0.09413)	(0.30824)
	[−1.45564]	[−0.70960]
D(DFERT(−2))	−0.116929	0.099902
	(0.07050)	(0.23087)
	[−1.65846]	[0.43272]
C	0.099981	0.036109
	(0.20901)	(0.68443)
	[0.47835]	[0.05276]
R-squared	0.627337	0.489819
Adj. R-squared	0.560790	0.398715
F-statistic	9.426997	5.376493
Log likelihood	−51.15158	−91.48200
Akaike AIC	3,.361858	5.734235
Schwarz SC	3.631215	6.003593

* denotes significant at 5% level.

The implication of the theoretical results is that if there is excess depletion or under utilization of ground water there will be scarcity of water and it will act as a constraint to future growth in agriculture. The econometric analysis establishes that there is meaningful relationship between (i) tube-well irrigation and productivity in agriculture, (ii) fertilizer use and productivity and (iii) tube-well irrigation and fertilizer use. There is no doubt that tube-well irrigation has been the main driving force in productivity growth in Indian agriculture through its effect on the use of HYV seeds and modern inputs. But in the recent years, the growth rate of tube-well irrigation has declined due to excess depletion or other reasons like lack of public investment and non-favourable natural conditions.

The annual average growth rate of yield in foodgrains production in India has declined to 1.11% in the period from 1995 to 2007 from 2.77% during the period from 1970 to 1995. The annual average growth rate of tube-well irrigation also has declined from 3.11% in the period from 1970 to 1995 to 1.24% in 1995-2007. Similarly, growth rate of fertilizer use has declined from 7.00% to 2.92% during the same period (See Sasmal, 2012). Thus the econometric results are found to be consistent with the theoretical analysis of the study. Here, the declining growth rate in yield has been associated with the declining growth rates of tube-well irrigation and fertilizer use. However, the declining growth rate of tube-well irrigation is not always due to excess depletion of ground water. The ground water has been over exploited in states like Punjab (145%), Haryana (109%) and Rajasthan (125%). The exploitation rate is very high in states like Uttar Pradesh (70%), Tamil Nadu (85%), Gujarat (76%) and Karnataka (70%). Here, 100% is the maximum permissible limit of extraction given the rainfall, recharge rate and soil conditions. There is under utilization of water in states like Bihar (39%), Assam (22%), Orissa (18%) and Jharkhand (20%) for various reasons. On the whole 58% of the ground water potential for irrigation in India has been utilized and there is scope for utilizing the remaining 42%. (Source: Ground Water Scenario of India, 2009-10). However, in the last few years, the annual average growth rate in yield has increased to 2%. This may be due to better management and utilization of surface water, technological change and other reasons like increase in public investment and higher productivity in rainfed agriculture.

4. The Summary

The LDCs have achieved remarkable success in foodgrains production in the last few decades and the tube-well irrigation has played a crucial role in this process. Irrigation has greatly facilitated the use of High-Yielding Variety (HYV) seeds, chemical fertilisers and other modern inputs to raise productivity in the farming sector. In India, the percentage share of well-irrigation in total irrigation has significantly increased in the last four decades and it has played a vital role in the productivity growth of foodgrains production in the country. However, rampant digging of tube-wells and excess depletion of ground water have put a question mark before the sustainability of growth. The private individuals make under valuation of natural resources leading to over exploitation of the resource. The price support, input subsidy of the government and public investment have encouraged excess depletion of ground water. The water scarcity is not always due to excess depletion. There are cases where water is under utilized due to many factors including lack of sufficient public investment.

This paper demonstrates theoretically how excess depletion or under utilization of ground water becomes a serious constraint to agricultural growth. The time series econometric analysis has been done using Indian data to have test of cointegration and vector error correction between the variables in VAR framework. The results show that the yield in foodgrains production, tube-well irrigation and fertilizer use are cointegrated implying that there is meaningful long-run relationship among them. The error correction estimation shows that the deviations from the long run relationship is adjusted by the error term and the variables in lags. The declining rate of growth in productivity is found to be associated with declining rates of growth of tube-well irrigation and fertilizer use. The decline in the growth rate of tube-well irrigation is not always due to excess depletion of ground water. Lack of public investment and appropriate technology and non-favourable natural conditions are also responsible for this is many cases.

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APPENDIX – A

Differentiation of (8) – (10) in Section 2 w.r.t. W, Z and S gives

$$\begin{bmatrix} P.F_{WW} - C_W^W - C_W^W - W \cdot C_{WW}^W & 0 & -C_S^W \\ P.F_{ZW} & P.F_{ZZ} & 0 \\ C_S^W & 0 & W \cdot C_{SS}^W \end{bmatrix}$$

$|D_1| < 0, \; |D_2| > 0, \; |D_3| < 0.$