2016

M.Sc. Part–II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—X (OR)

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper

[Advanced Optimization and Operations Research–II]

Answer Q. No. *11* and any *six* from the rest.

[Calculator may be used]

1. (a) In a certain community 25% of all girls are blondes and 75% of all blondes have blue eyes. Also, 50% of all girls in the community have blue eyes. If you know

that a girl has blue eyes, how much additional information do you get by being informed that she is blonde ? 4

(b) Define entropy and show that it is continuous. 4

(c) Define joint and conditional entropies. Prove that $H(X, Y) \leq H(X) + H(Y)$, with equality iff X and Y are independent. 8

2. (a) What is simulation ? Describe its advantages in solving the problems. Give its main limitations. 4

(b) Explain Monte-Carlo simulation. State different mathematical steps in Monte-Carlo method. 4

(c) The management of bank in thinking of opening a drive in facility for its branch office in a commercial area. The inter-arrival times of the customers at the branch are as follows :

Inter-arrival time (minutes)	Probability
3	0.17
4	0.25
5	0.25
6	0.20
7	0.13

It is planned to have one cashier who can serve the customers at the following rate :

Service time (minutes)	Probability
3	0.10
4	0.30
5	0.40
6	0.15
7	0.05

Determine the number of spaces to be planned for waiting cars. Simulate the operation of the facility for arriving sample of 25 cars. If the location has space for not more than two waiting cars how many customers would be turned away due to lack of space ? What is the average waiting time of a customer ? 8

3. (a) Obtain the probability p_n of n customers in the system for the (M/M/C) : (∞/FCFS/∞) queuing model.

(b) Describe different type of service decipline.
Define the following terms :
mean servicing rate, mean arrival rate and traffic intensity. 10+6

4. (a) What is network analysis ? What are the advantages of it ?

Explain the terms :

total float, independent float, free float, slack and crashing. 3+5

(b) A project consists of eight activities with the following relevant information :

Activity	Immediate Predecessor	Estimated duration (days)		
		Optimistic	Most likely	Pessimistic
A	—	3	6	9
B	—	2	5	8
C	A	2	4	6
D	B	3	4	11
E	B	1	3	3
F	C, D	2	5	8
G	E	1	5	15

(i) Draw the network and find out the expected project completion time.

(ii) What duration will have 25% confidence for project completion ? 8

5. (a) Find the optimum order level for a product for which the price breaks are as follows :

Range of quantity to be purchased	Unit purchase cost
$b_0 \leq Q < b_1$	p_1
$b_1 \leq Q < b_2$	p_2
$b_2 \leq Q < b_3$	p_3

Here

(i) demand rate is known and uniform

(ii) shortages are not permitted

(iii) production for supply of commodities is instantaneous

(iv) lead time is zero.

(b) Find the optimum order quantity for a product for which the price breaks are as follows :

Quantity	Purchase cost (per unit)
$0 \leq Q_1 < 100$	Rs. 20
$100 \leq Q_2 < 200$	Rs. 18
$200 \leq Q_3$	Rs. 16

The monthly demand for the product is 400 units. The storage cost is 20% of the unit cost of the product and the cost of ordering is Rs. 25 per month. 8+8

6. (a) Describe dynamic programming method to solve the following problem :

Minimize $z = f_1(y) + f_2(y_2) + \ldots\ldots + f_n(y_n)$
subject to $y_1 y_2 \ldots y_n \geq p$, $p > 0$, $y_j \geq 0$ for all j.

Use this technique to solve the following problem :

Minimize $z = y_1 + y_2 + \ldots\ldots + y_n$
subject to $y_1 y_2 \ldots\ldots y_n = a$, $y_j \geq 0$ for all j. 8

(b) Solve the following LPP using dynamic programming method :

Maximize $z = 3x_1 + 5x_2$
subject to
$x_1 \leq 4$
$x_2 \leq 6$
$3x_1 + 2x_2 \leq 18$
$x_1, x_2 \geq 0$ 8

7. (a) How do you solve the following geometric programming problem :
Find $x = (x_1, x_2, \ldots x_n)^T$ that minimizes the objective function

$$f(x) = \sum_{j=1}^{N} U_j(x) = \sum_{j=1}^{N} C_j \prod_{i=1}^{n} x_i^{a_{ij}}$$

where $C_j > 0$, $x_i > 0$ and a_{ij} are real constants. 8

(b) Define the reliability of a system. What are MTBF and MTTF ? Show that $R(t) = \exp\left[-\int_0^t \lambda(t)\,dt\right]$, where R(t) is the reliability function and $\lambda(t)$ represents the failure rate. 8

8. (a) At time zero, all items in a certain system are new. Each item has a probability p of falling immediately before the end of the first month of life and a probability 1 – p(= q) of failing immediately before the end of the second month. If all items are replaced as they fail. Show that the expected number of failures f(x) at the end of the month x is

$$f(x) = \frac{N}{1+q}\left[1-(-q)^{x+1}\right]$$

N being the number of items in the system. Hence find the number of replacement under individual policy. 8

(b) Find the optimal control u and the optimal path that extremizes

$$J = \frac{1}{2}\int_0^{\infty} \left(x^2 + 4u^2\right) dt$$

where $\ddot{x} = -\dot{x} + u$ and $x(0) = a$, $\dot{x}(0) = b$ both x and $\dot{x} \to 0$ as $t \to \infty$. 8

9. (a) Derive Johnson's algorithm for processing n jobs through three machines in a sequence.

What is sequencing problem? Explain the term 'elapsed time'. 6+2

(b) What is replacement? Deduce the optimal replacement policy(s) for items whose running cost increases with time in discrete units and value of money remains constant during a period. 8

10. (a) Define a non-cooperative game with n players. What is equilibrium situation? When two games are said to be strategically equivalent? Show that strategically equivalent games obey symmetric and transistive properties. 8

(b) Find an optimal sequence for following sequencing problem of four jobs and five machines, when passing is not allowed. Its processing time (in hours) is given below:

JOB	Machine				
	M_1	M_2	M_3	M_4	M_5
A	7	5	2	3	9
B	6	6	4	5	10
C	5	4	5	6	8
D	8	3	3	2	6

Also find the total elapsed time. 8

11. (a) Define the following terms :
Lead time and planning horizon.

(b) Define the degree of difficulty in connection with geometric programming. 2

2016

M.Sc. Part–II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—X (OM)

Full Marks : 75

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper

Answer any *five* questions. 5×15

1. (a) Derive the equivalence property of Emagram and discuss its different properties. 7

(b) Deduce the equation of state for moist air in the atmosphere in the following form

$$p\alpha = R_d T\left[1-(1-\epsilon)\frac{e}{p}\right],$$

the symbols have their usual meanings. 5

(c) Show that for an isentropic process, $p\alpha^{\gamma}$ = constant. 3

2. (a) Discuss about the phase change of an ideal gas and derive the relation of dependency of latent heat of evaporation with respect to temperature during the phase change of an air parcel. 7

(b) Define specific entropy and establish the relationship between the specific entropy and the potential temperature. 5

(c) Define virtual temperature and show that if T_v is the virtual temperature, then $T_v = T(1 + 0.61r)$ where r is the mixing ration. 3

3. (a) Define homogeneous atmosphere. Show that the height of the homogeneous atmosphere depends entirely on the temperature at the bottom. Also prove that the pressure at the top of the homogeneous isothermal atmosphere is equal to $\frac{1}{e}$ times that at the sea level. 7

(b) Derive the pressure tendency below a frontal surface in the atmosphere. 5

(c) Derive the geostrophic wind equation in the atmosphere. 3

4. (a) How is the thermal wind formed in the atmosphere? Derive the thermal wind components in the atmosphere. 7

(b) What do you mean by isobaric cooling? Show that the relative increase in dew-point temperature is about 5% of the sum of relative increase in mixing ratio and pressure. 5

(c) Discuss different types of fronts in the atmosphere. 3

5. (a) Derive the horizontal equation of motion of an air parcel in the atmosphere in natural co-ordinate system. 7

(b) Derive the expression of water vapor content in an air column in the atmosphere. 5

(c) Discuss different types of fronts in the atmosphere. 3

6. (a) Derive the effect of ascent and descent of an air parcel on lapse rate in terms of pressure changes. 7

(b) Discuss the different cases of pressure changes in the atmosphere with respect to altitude. 5

(c) What is lifting condensation level? 2

(d) Define relative humidity. 1

7. (a) What is turbulent motion ? Explain turbulent transfer of momentum in the atmosphere. 7

(b) Derive the expression of the pressure gradient force in the atmosphere. 4

(c) Derive the adiabatic lapse rate of unsaturated moist air. 4

8. (a) Show that during the adiabatic process of an saturated air parcel in the atmosphere, the relation between equivalent potential temperature and potential temperature of the air parcel is

$$\theta_e = \theta_e^{\frac{Lw_s}{C_p T}}$$

where all symbols have their usual meanings. 7

(b) Derive the adiabatic lapse rate of unsaturated moist air. 4

(c) Derive the hypsometric equation in the atmosphere. 4

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