2016

M.Sc. Part-II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—IX (OR/OM)

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper : OR

Answer Q. No. *11* and any *six* from the rest.

1. (a) Briefly describe Wolfe's algorithm for solving a quadratic programming problem. 8

   (b) State and prove Fritz-John stationary point necessary optimality theorem. 8

2. (a) Let θ be a numerical function on an open convex set $\Gamma \subset R^n$. A necessary and sufficient condition that θ be convex on Γ is that for each $x^1, x^2 \in \Gamma$. Prove that

$$\left[\nabla\theta(x^2) - \nabla\theta(x^1)\right](x^2 - x^1) \geqq 0 .$$ 6

(b) State and prove Motzkin's theorem of the alternative in connection with non-linear programming. 5

(c) State Farkas' theorem and give its geometrical interpretation of non-linear programming. 5

3. (a) Using decomposition principle, reduce the following problem to an elegant form of LPP which can be solved by simplex method.

$$\begin{aligned} \text{Minimize } z &= -x_1 - x_2 - 2y_1 - y_2 \\ \text{subject to } x_1 + 2x_2 + 2y_1 + y_2 &\le 40 \\ x_1 + 3x_2 &\le 30 \\ 2x_1 + x_2 &\le 20 \\ y_1 &\le 10 \\ y_2 &\le 10 \\ y_1 + y_2 &\le 15 \\ x_j, y_j \ge 0, j &= 1, 2. \end{aligned}$$ 8

(b) Define

(i) Minimization problem involving differentiable function.

(ii) Fritz-John stationary point problem

(iii) Kuhn-Tucker stationary point

(iv) Differentiable convex function. 4×2

4. (a) Use the artificial constraint method to find the initial basic solution of the following problem and then apply the dual simplex algorithm to solve it

$$\begin{aligned} \text{Maximize } z &= x_1 - 3x_2 - 2x_3 \\ \text{subject to } x_2 - 2x_3 &\ge 2 \\ x_1 - 4x_2 - 6x_3 &= 8 \\ 2x_2 + x_3 &\le 5 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$ 8

(b) Solve by Beale's method

$$\begin{aligned} \text{Minimize } z &= 183 - 44x_1 - 42x_2 + 8x_1^2 - 12x_1x_2 + 17x_2^2 \\ \text{subject to } 2x_1 + x_2 &\le 10 \\ x_1, x_2 &\ge 0 \end{aligned}$$ 8

5. (a) State and prove Fritz-John saddle point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of Fritz-John saddle point theorem. 8

(b) Solve the quadratic programming problem using Wolfe's method

$$\text{Minimize } f(x) = x_1^2 + x_2^2 - 2x_1 - 4x_2$$

$$\text{subject to } \quad x_1 + 4x_2 \le 5$$
$$2x_1 + 3x_2 \le 6$$
$$x_1, x_2 \ge 0$$

8

6. (a) Discuss the effect of discrete change in the requirement vector of the optimal simplex table of the following LPP :

$$\text{Maximize } z = x_1 - x_2 + 3x_3$$

subject to the constraints

$$x_1 + x_2 + x_3 \le 10$$
$$2x_1 - x_3 \le 2$$
$$2x_1 - 2x_2 + 3x_3 \le 0$$
$$x_1, x_2, x_3 \ge 0$$

so that the optimality of this problem remains undisturbed. The optimal simplex table is given below :

C_B	Basis	X_B	y_1	y_2	y_3	y_4	y_5	y_6
−1	x_2	6	$\frac{1}{5}$	1	0	$\frac{3}{5}$	0	$-\frac{1}{5}$
0	x_5	6	$\frac{14}{5}$	0	0	$\frac{2}{5}$	1	$\frac{1}{5}$
3	x_3	6	$\frac{4}{5}$	0	1	$\frac{2}{5}$	0	$\frac{1}{5}$
		$z = 6$	$\frac{6}{5}$	0	0	$\frac{3}{5}$	0	$\frac{4}{5}$

8

(b) Solve the following IPP by Gomory's cutting method

$$\text{Maximize } z = x_1 + 4x_2$$

$$\text{subject to } 2x_1 + 4x_2 \le 7$$
$$5x_1 + 3x_2 \le 15$$
$$x_1, x_2 \ge 0 \text{ and integers}$$

8

7. (a) Find the range of discrete changes of cost vector of the LPP

$$\text{Maximize } Z = CX$$
$$\text{subject to } AX = b$$
$$X \ge 0$$

so that the optimality of this problem remains unchange. 8

(b) Solve the following problem by cutting plane method :

$$\text{Maximize } f(x_1, x_2) = 1 - 4x_1 - 2x_2$$

$$\text{subject to } 2(x_1 - 2)^2 + (x_2 - 3)^2 \le 12$$
$$2x_1 + x_2 \le 3$$
$$0 \le x_1, x_2 \le 5$$

with tollerance $\in = 0.2$ 8

8. (a) What are the limitations of Fibonacci method ? Write down the steps of Fibonacci method to optimize a function of single variable. 2+6

(b) Minimize the function $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 + 2gx_1 + 2fx_2 + 2hx_3 + c$ using steepest descent method starting from the point $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$. 8

9. (a) Solve the following GPP graphically :

Minimize $Z = P_1(d_2^- + 2d_3^-) + P_2d_6^+ + P_3d_6^-$

subject to the constraints

$20x_1 + 10x_2 + d_4^- - d_4^+ = 60$

$10x_1 + 10x_2 + d_5^- - d_5^+ = 40$

$40x_1 + 80x_2 + d_1^- - d_1^+ = 1000$

$x_1 + d_2^- - d_2^+ = 4$

$x_2 + d_3^- - d_3^+ = 6$

$d_4^+ + d_5^+ + d_6^- - d_6^+ = 50$

$x_1, x_2, d_i^-, d_i^+ \geq 0, \ i = 1, 2, 3, 4, 5, 6$ 8

(b) Using revised simplex method solve the following LPP :

Maximize $Z = x_1 + 2x_2$

subject to $2x_1 + 5x_2 \geq 6$

$x_1 + x_2 \geq 2$

$x_1, x_2 \geq 0$ 8

10. (a) Discuss how to compute $\hat{B}^{-1}$ in revised simplex method. What are the advantages of revised simplex method ? 6+2

(b) Minimize $f(x) = \begin{cases} \dfrac{(x^2 - 6x + 13)}{4}, & x \leq 4 \\ x - 2, & x > 4 \end{cases}$

in the interval [2, 6] by Fibonacci method taking the number of experiments (n) = 6. 8

11. Answer any *one* : 4×1

(a) Discuss how the "Branch and Bound" method worked. 4

(b) Write a short note on quadratic programming. 4

Special Paper : OM

Answer Q. No. *1* and any *six* from the rest.

1. Answer any *two* questions : 2×2

   (a) Write the continuity equation for Quasi-Geostrophic flow.

   (b) Define the Rossby number and discuss its physical significance for the case of its smaller value.

   (c) Define the Reynolds number and discuss its physical significance for the case of its higher value.

2. (a) Derive the equation of continuity for compressible flow in conservative form and then reduce to its non-conservative form. Also write the equation for steady flow. 8+2

   (b) An incompressible velocity field is given by $u = a(x^2 - y^2)$, v is unknown, w = b where a and b are constants. What must be the form of velocity v be ? 6

3. For the Sverdrup waves for a long surface waves in a rotating ocean of unlimited horizontal extent, find the phase speed and group velocity. Show that the horizontal

velocity vector describes an ellipsis where the ratio of the major axis to the minor axis is $|\omega / f|$ where notations have their usual meaning. 8+8

4. Write the total velocities in terms of Geostrophic and Ageostrophic components and hence under the necessary assumptions deduce the equations for Geostrophic balance as

$$v_g = \frac{1}{f_0}\frac{\partial \varphi}{\partial x},$$

$$u_g = -\frac{1}{f_0}\frac{\partial \varphi}{\partial y}$$

where symbols have their usual meanings. 16

5. (a) Establish Gibb's relation of thermodynamics. Deduce Gibb's-Duhem relation. 8

   (b) Define salinity (s) and concentration of pure water (c_w) and hence show that $s + c_w = 1$. 8

6. Deduce the equations of motion under long-wave theory. 16

7. Deduce the momentum equation of motion of the fluid on ratating earth. Also explain the physical interpretation of each term. 11+5

8. Deduce the equation of conservation of energy of sea-water taken as mixture of salt and pure water. 16

9. Derive the governing equations of motion of sea-water under Boussinesq appromixations' to be stated by you. 16

10. Define Rossby number. Deduce the governing equations of thermal wind when Rossby number is small. Hence deduce the Taylor-Proudman theorem. 3+10+3

———