2016

M.Sc. Part–II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—VII

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

[Marks : 25]

Answer Q. No. *1* and any *two* from the rest.

1. What is Pynting vector ? Give its physical meaning. 1

2. Define electrical image. Find the image of a single point charge 'e' at a distance 'f' from a conducting infinite plane sheet kept at zero potential. Deduce the expression for the surface density of induced charge in this case. 2+4+6

3. (a) Prove the following Maxwell's equation

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

where symbols used have their usual meaning.

(b) Deduce equation of continuity from the law of conservation of energy. 8+4

4. What do you mean by Lienard wiechert potentials ? What is the significance of Lienard wiechert potential ? Derive the Lienard wiechert potential for a moving point charge. 2+3+7

Group—B

(Fuzzy Sets and its application in O.R.)

[Marks : 25]

Answer Q. No. 5 and any *three* from Q. No. 6 to Q. No. 10.

5. Answer any *one* question : 1

(a) Write some applications of fuzzy set theory.

(b) Define the intersection of two fuzzy sets.

6. (a) What are the causes of uncertainity ? Explain the traditional and modern view of uncertainity with examples. 4

(b) If $\tilde{A} = [a_1, a_2, a_3]$ and $\tilde{B} = [b_1, b_2, b_3]$ be two triangular fuzzy numbers, then prove that

$$\tilde{A} + \tilde{B} = [a_1 + b_1, a_2 + b_2, a_3 + b_3].$$ 4

7. Analytically prove the distributive law

$$\tilde{A} \cap (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C})$$

using definition of union and intersection of fuzzy sets. 8

8. Let the membership functions of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ be as follows

$$\mu_A(x) = \begin{cases} 0 & \text{if} \quad x \le 1 \\ (x-1)/2 & \text{if} \quad 1 < x \le 3 \\ (6-x)/3 & \text{if} \quad 3 < x < 6 \\ 0 & \text{if} \quad x \ge 6 \end{cases}$$

$$\underset{\sim}{\mu_B}(x) = \begin{cases} 0 & \text{if} \quad x \le 3 \\ (x-3)/3 & \text{if} \quad 3 < x \le 6 \\ (8-x)/2 & \text{if} \quad 6 < x < 8 \\ 0 & \text{if} \quad x \ge 8 \end{cases}$$

Determine the membership function of

$\underset{\sim}{A} \cup \underset{\sim}{B}$, $\underset{\sim}{A} \cap \underset{\sim}{B}$, $\underset{\sim}{A}^c$ and $\underset{\sim}{B}^c$. 2+2+2+2

9. Using Wernor's method slove the following fuzzy LPP :

Max $z = 2x_1 + x_2$

subject to $x_1 \underset{\sim}{\le} 3$

$x_1 + x_2 \underset{\sim}{\le} 4$

$0.5x_1 + x_2 \le 3$; $x_1, x_2 \ge 0$ with

to lerances as $p_0 = 1, p_1 = 2, p_2 = 2$. 8

10. (a) Discuss Verdegay's method to solve a fuzzy LPP. 6

(b) Using Zadeh's extension principle, show that $[4, 8] - [3, 5] = [-1, 5]$. 2

Group—C

[Marks : 30]

11. Answer any *two* questions : 2×3

(a) **Define all three components of vorticity in terms of velocities and find the values for irrotational flow.** 3

(b) **If a velocity potential exists for the velocity field of the form :**

$$u = a(x^2 - y^2) \qquad v = -2axy \qquad w = 0$$

find it. 3

(c) **Explain D'Alembert's paradox.** 3

12. Answer any *three* questions : 3×8

(a) **Derive the substantial derivative for an infinitesimally small fluid element moving with the flow and discuss the physical significance of substantial derivative of temperature.** 8

(b) **Assuming the necessary stress-strain rate relations, deduce Navier-Stokes equations of motion of an incompressible viscous fluid.** 8

(c) **Deduce the Prandtl's boundary layer equations in two dimensional flows.** 8

(d) Define vortex rows. Show that the motion due to a set of line vortices of strength k at points $z = \pm na$, $(n = 0, 1, 2, \ldots)$ is given by the relation

$$w = \frac{ik}{2\pi} \log\left\{\sin\left(\frac{\pi z}{a}\right)\right\}.$$ 2+6

(e) State and prove Kutta-Joukowski's theorem. 8

Group—D

(Magnetohydrodynamics)

[Marks : 20]

Answer any *two* questions : 2×10

13. A viscous, incompressible conducting fluid of uniform density is confined between a channel made by an infinitely long conducting horizontal plate $y = -L$ and a horizontal infinite non-conducting plate $y = L$ (upper). Assume that there is no pressure gradient and a uniform magnetic field H_0 acts perpendicular to the plates. Both the plates are at rest. Find the velocity of the fluid and the magnetic field. 10

14. State and prove Alfven's theorem. 10

15. (a) Deduce the equation of motion of a conducting fluid.

(b) Prove that the rate of change of magnetic energy is the sum of loss of magnetic energy per unit volume and the work done by the material against the force exerted by the magnetic field on the currents during motion.

5+5

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