2016

M.Sc. Part–I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—V

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

(*Mechanics of Continuous Media*)

Marks : 50

Answer *Q. No. 6* and any *three* questions from the rest.

1. (a) State and prove the Cauchy's first equation of motion. When the continuum is in static equilibrium ? Deduce the equation of equilibrium. 8

(b) The strain tensor at a point is given by :

$$(E_{ij}) = \begin{pmatrix} 5 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

Determine the extension of line element in the direction of $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$. What is the change of angle between two perpendicular line elements in the directions of $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ and $\left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right)$? 4

(c) Find the image of a source of strength m with respect to a circle. 4

2. (a) What is the concept of strain deformation ? Derive the change in the length of any line element in a continuum by material method. 8

(b) The stress matrix at a point $P(x_i)$ in a continuum is given by : 4

$$(T_{ij}) = \begin{pmatrix} x_1x_3 & x_3^2 & 0 \\ x_3^2 & 0 & -x_2 \\ 0 & -x_2 & 0 \end{pmatrix}$$

Find the stress vector at a point Q(1, 0, –1) on the surface $2x_1^3 = x_2^2 + x_3^2$.

(c) Find the stream line and path line of a fluid motion for the velocity field

$$u = \frac{x}{1+t},\ v = y,\ w = 0.$$ 4

3. (a) Show that of all possible motions of a homogeneous liquid the irrotational motion has minimum kinetic energy if the normal velocity of the boundary is prescribed. 8

(b) Define strain quadric. Prove that the extension of a line element through the centre of strain quadric in the direction of any central radius vector is equal to the inverse of the square of the radius vector. 4

(c) Establish the stress vector – stress tensor relationship. 4

4. (a) Two identical closed cylinders of height C with their bases on the same horizontal plane are filled, one with water and the other with air of such density as to support a column h of water, $h < c$. If a communication be opened between them at their bases, the height x to which water rises is given by the equation :

$$cx - x^2 + ch\log\left(\frac{c-x}{c}\right) = 0.$$ 8

(b) Show that

$$u = -\frac{2xyz}{\left(x^2+y^2\right)^2},\quad v = -\frac{\left(x^2-y^2\right)z}{\left(x^2+y^2\right)^2},\quad w = \frac{y}{x^2+y^2}$$

are the velocity components of a possible liquid motion. Is this motion irrotational? 4

(c) Show that the following are not possible strain components :

$$E_{11} = K\left(x_1^2 + x_2^2\right),\ E_{22} = K\left(x_2^2 + x_3^2\right),\ E_{12} = K'x_1x_2x_3,$$

$$E_{13} = E_{23} = E_{33} = 0$$

where K and K′ are constant. 4

5. (a) Prove that the stress vector at a point on any arbitrary plane surface is a linear function of three stress vectors acting on any three mutually perpendicular planes through that point. 8

(b) State and prove Milne Thomson Circle theorem. Applying this theorem, find the velocity potential at a point in fluid when a uniform stream is flowing past a fixed circular cylinder of radius a. 8

6. State Generalized Hooke's law. How many elastic modulii are involved in their law? 2

or

Define irrotational motion and vortex motion. 2
