2016

M.Sc. Part-I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—IV

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answer to questions of each group in Separate answer booklet.

Group—A

(*Principles of Mechanics*)

[Marks : 50]

Answer *Q. No. 1* and any *three* questions from the rest.

1. Answer any *one* question : 2

(a) What do you mean by generalised force? Find the expression of it in terms of generalised coordinates.

(b) Show that the generalised momentum corresponding to a cyclic co-ordinate is a constant of motion.

2. (a) What do you mean by Euler angles ? Suppose a rigid body is rotating about a fixed point. Deduce the relation between the coordinates (x, y, z) (in fixed set of axes) and (x′, y′, z′) (in rotating set of axes), in terms of Euler angles. 8

(b) Derive the following differential operator :

$$\left(\frac{d\bar{R}}{dt}\right)_{fix} = \left(\frac{d\bar{R}}{dt}\right)_{rot} + w \times \bar{R}$$

where $\left(\frac{d\bar{R}}{dt}\right)_{fix}$ and $\left(\frac{d\bar{R}}{dt}\right)_{rot}$ represent the derivative of any vector $\bar{R}$ with respect to fixed and rotating frame of axes respectively. 8

3. (a) Deduce Routhian equations of motion for n generalized coordinates and K ignorable coordinates.

Hence prove that, if all the coordinates of a dynamical system of n degrees of freedom are ignorable then the problem can be solved completely by integration. 8

(b) Obtain Hamilton's equations of motion where the Hamiltonian is given by :

$$H = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$$

Also, show that

q_1q_2 = constant, $(p_2 - bq_2) / q_1$ = constant, a, b are constant. 8

4. (a) Given $J = \int_{x_0}^{x_1} F(y, y', x)\,dx$, where F is a known function, y depends on x but unknown. Show that y can be determined from the equation :

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial y} = 0$$

such that J is an extremum. 8

(b) Deduce the Hamilton's equations of motion for holonomic and conservative system from Hamilton's principle. 8

5. (a) Test whether the following is a Canonical transformation : 8

$$p_1 = \frac{1}{2}\sqrt{2Q_1}\,\sin P_1 + \frac{1}{2}\sqrt{2Q_2}\,\sin P_2$$

$$q_1 = \sqrt{2Q_1}\,\cos P_1 + \sqrt{2Q_2}\,\cos P_2$$

$$p_2 = -\frac{1}{2}\sqrt{2Q_1}\,\sin P_1 + \frac{1}{2}\sqrt{2Q_2}\,\sin P_2$$

$$q_2 = -\sqrt{2Q_1}\,\cos P_1 + \sqrt{2Q_2}\,\cos P_2.$$

(b) Define Canonical transformation. Show that the transformation (q, p, t) → (Q, P, t) is canonical if $[Q, P]_{(q,\,p)} = 1$. 4

(c) Show that the Hamiltonian is the total energy when the system is moving in a conservative force field in a sceleronomous system. 4

6. (a) In relativistic mechanics, show that the mass of a particle increases with velocity. 8

(b) Consider the equilibrium configuration of the molecule such that two of its atoms of each of mass M are symmetrically placed on each side of the third atom of mass m. All the three atoms are collinear. Assume the motion along the line of molecules and there being no interaction between the end atoms. Compute the kinetic energy and potential energy of the system and discuss the motion of the atoms. 8

Group—B

(*Partial Differential Equation*)

[Marks : 50]

Answer *Q. No. 1* and any *three* from the rest.

1. (a) What is the geometric significance of the first order PDE ? 2

Or

(b) What do you mean by well-posed problem in partial differential equation ? 2

2. (a) Show that Lagrange equation :

$$(2z-y)p + (x+z)q + (2x+y) = 0$$

has the complete integral

$$x^2 + y^2 + z^2 = a(x - 2y - z) + b,$$

a family of sphere.

Find the envelope of the one parameter family by substituting $b = 1 - \frac{3a^2}{2}$ and show that the envelope is a part of the given integral. 8

(b) Solve : $D(D-2D')(D+D')Z = e^{x+2y}\left(x^2+4y^2\right)$

where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$. 8

3. (a) Find the complete integral of the following equations :

(i) $q = (z+px)^2$;

(ii) $px + qy = pq$. 4+4

(b) Solve by Charpit's method the partial differential equation : 8

$$2x\left[z^2\left(\frac{\partial z}{\partial y}\right)^2 + 1\right\} = z\left(\frac{\partial z}{\partial x}\right)$$

4. (a) Consider a PDE of the form : 8

$Rr + Ss + Tt + f(x, y, z, p, q) = 0$ ..... (1)

where R, r, S, s, T, t, f, p, q have their usual meaning. With the help of following transformation :

$$\xi = \xi(x,y) \text{ and } \eta = \eta(x,y)$$

and necessary assumptions, reduce the above equation (1) to its canonical form.

(b) Consider the equation : 8

$$y^2u_{xx} - x^2u_{yy} = 0, \quad \left.\begin{matrix} x>0 \\ y>0 \end{matrix}\right\}$$

What is the nature of the equation in first quadrant ? Find the new characteristic co-ordinates that will change the original equation to canonical form for x, y in first quadrant. Find the canonical form.

5. (a) Solve the diffusion equation : 10

$$\frac{\partial T}{\partial \lambda} = \frac{\partial^2 T}{\partial \lambda^2}$$

satisfying the boundary conditions :

T = 0 at λ = 0 and 1,

and initial condition :

$$T(\lambda, 0) = \begin{cases} 2\lambda, & 0 \le \lambda \le \frac{1}{2} \\ 2(1-\lambda), & \frac{1}{2} \le \lambda \le 1 \end{cases}$$

(b) Show that for a two dimensional and simply connected region G, it is possible to reduce Neumann problem to Dirichlet problem. 6

6. (a) Construct two adjoint of the differential equation : $L(Z) = C^2 Z_{\lambda\lambda} - Z_{AA}$. 6

(b) Solve the Laplace's equation :

$$\nabla^2 \phi = 0,$$

in the semi-infinite region $\lambda \ge 0$, $0 \le y \le 1$ subject to the boundary conditions $\phi_x(0, y) = \phi_y(\lambda, 0) = 0$, $\phi(x, 1) = f(x)$. 10

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