2016

M.Sc. Part–I Examination

APPLIED MATHEMATICS WITH
OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—III

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answer to questions of each group in Separate answer booklet.

Group—A

(Probability and Statistics)

[Marks : 30]

Answer any *two* questions :   2×15

1. (a) Describe a linear birth and death process and derive the differential equations describing the process. Also find the mean population size under this process.

10

(b) Deduce the equation of the plane of regression containing three variables. 5

2. (a) What is the concept of probability distribution associated with Markov chain ?

Let $\{X_n, n \geq 0\}$ be a Markov chain having state space $S = \{1,2,3,4\}$ and corresponding transition probability matrix :

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Show that the state three and four are transient, the state two is persistent, and the state one is ergodic. 8

(b) Deduce the differential equations (forward and backward diffusion equations) for a Wiener process. 7

3. (a) State and prove first entrance theorem. 4

(b) Explain the terms : 6

   (i) Ergodic process ;

   (ii) Closed set of State ;

   (iii) Transient State ;

   (iv) Persistent State.

(c) What do you mean by Markov chain and order of Markov chain in Stochastic process ? Define Markov process. 5

Group—B

(Numerical Analysis)

[Marks : 40]

Answer Q. No. 4 and any *three* from the rest.

4. Prove the following relations : 4

(a) $\mu^2 f(x) = \frac{1}{4}\left[\delta^2 + 4\right] f(x)$.

(b) $hD\ f(x) = \sin h^{-1}(\mu\ \delta f(x))$,

where μ, δ and D represent average operator, central difference operator, and differential operator. h is the step length.

5. Describe Lagrange's inverse interpolation method.

Use this method to find the value of x when y = 8.2 from the following table :

x :	0	1	2	5
y :	5	8	20	35

6+6

6. Describe Gauss-Chebyshev 3-point quadrature formula.

Use this method to find the value of $\int_0^1 \dfrac{x^2}{1+x^2}\,dx$ correct upto four decimal places. 6+6

7. (a) Use Chebyshev polynomials to find least squares approximation of second degree for $f(x) = \sqrt{1-x^2}$ on $[-1, 1]$. 5

(b) Discuss Milne-Predictor-Corrector method to solve the following differential equation :

$$\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0.$$ 7

8. (a) Discuss the stability of the second order Runga-Kutta method. 4

(b) Explain Power method to find the largest eigenvalue and the corresponding eigenvector of a matrix. Is the method applicable to find smallest eigenvalue of a matrix ? Justify your answer. 8

9. (a) Describe an explicit method to solve the following wave equation :

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2},\ t \geq 0,\ 0<x<1,\ C \text{ is a real constant,}$$

with the initial conditions u(x, 0) = f(x) and

$$\left.\frac{\partial u}{\partial t}\right)_{(x,0)} = g(x),\ 0<x<1$$

and the boundary conditions

u(0, t) = ϕ (t) and u(1, t) = C (t), t > 0. 6

(b) Use Gaussian elimination with partial pivoting method to find the inverse of the following matrix : 6

$$\begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 5 \\ 2 & 3 & 4 \end{bmatrix}$$

Group—C

(*Introduction to Computing*)

[Marks : 30]

10. Answer any *six* questions : 6×5

(i) Write a program in C to find the prime factors of a positive number using a function.

(ii) How can the 'getchar' and 'putchar' functions be used to read and write a string ?

(iii) Using Karnaugh map, simplify the following function :

$$F(x, y, z, w) = \sum(1,3,5,8,9,11,15) + \sum(2,13)$$

(iv) What do you understand by normalised floating point representation of a real number ?

Evaluate $0.4568 \times 10^8 - 0.1234 \times 10^3$ using it.

(v) Write a program in C to find the sum of all digits in a number using a function.

(vi) (a) Substract $(1001001)_2$ from $(10010110)_2$ using 1's complement method of binary subtraction.

(b) Differentiate between encoding and conversion of a number.

(vii) A combinational circuit has four inputs and one output. The output is equal to 1 when (a) all the inputs are equal to 1 or (b) none of the inputs are equal to 1 or (c) an odd number of inputs are equal to 1.

Obtain the truth table. Find the simplified output function in sum of products.

(viii) Explain nested if-else and switch statements with an appropriate example for each.

(ix) Explain different types of storage class available in C programming language.

(x) What is structure ? How does a structure differ from an array ? How can structure variables be declared ? How are the members of structure variables assigned initial values ? How is a structure member accessed ?