2016

M.Sc.

3rd Semester Examination

PHYSICS

PAPER—PHS-301

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer-scripts for Group-A & Group-B

Group-A

(*Quantum Mechanics*)

[Marks : 20]

Answer *Q. No. 1 & 2* and any *one* from the rest.

1. Answer any *three* bits : 2×3

(a) Consider a system of three non-interacting identical $\frac{1}{2}$ spin particles that are in the same spin state $\left|\frac{1}{2}, \frac{1}{2}\right\rangle$ and confined to move in a one-dimensional infinite potential well of length a : $V(x) = 0$ for $0 < x < a$ and $V(x) = \infty$ for other value of x. Find the energy and wave function of the second excited state.

(b) The ground state of $_{13}Al^{27}$ is $^2P_{1/2}$. Under the action of a strong magnetic field the ground state energy level will split up into — levels. Draw the energy level diagram.

(c) State and prove optical theorem for δu scattering.

(d) Consider two identical linear harmonic oscillators, each of mass m and frequency w having interaction potential $\lambda x_1 \dot{x}_2$, where x_1 and x_2 are oscillator variables. Find the energy levels.

(e) Write down Lippman-Schwinger equation.

2. Answer any *one* bit : 1×4

(a) In a scattering experiment, the potential is spherically symmetric and the particles are scattered at such energy that only s and p waves need be considered.

$$\frac{d\sigma}{d\Omega} = a + b\cos\theta + c\cos^2\theta$$

Prove that total cross-section

$$\sigma = 4\pi\left(a + \frac{c}{3}\right).$$

(b) Consider a system of two spin half particles in a state with total spin quantum number S = 0. Find the eigen value of the spin Hamiltonian $\hat{H} = A\,\vec{S}_1.\vec{S}_2$, where A is a positive constant in this state.

3. (a) Two identical bosons, each of mass m, move in the one-dimensional harmonic potential $V(x) = \frac{1}{2} m w^2 x^2$. They also interact with each other via the potential

$$V_{int.} = \alpha \exp\left[-\beta (x_1 - x_2)^2\right]$$

where α and β are positive parameters.
Show that ground state energy of the system is

$$E_0 = \hbar w + \frac{m w \alpha}{\pi \hbar} \frac{1}{\sqrt{\left(\frac{mw}{\hbar}\right) + 2\beta}}.$$

$$\left[\text{Given : } \psi_0(x) = \left(\frac{mw}{\pi \hbar}\right)^{1/4} \exp\left(-\frac{m w x^2}{2\hbar}\right)\right]$$

(b) Show that for Yukawa $V(r) = V_0 \exp(-\alpha r)/r$, where V_0 and α are constants ;

$$\frac{d\sigma}{d\Omega} = \frac{4\mu^2 V_0^2}{\hbar^4 \alpha^4} \text{ if } E \to 0.$$

(c) Explain Ramsauer-Townsend effect. 5+3+2

4. (a) Find an expression for the electron density n(r) in the Thomas-Fermi model in terms of the Thomas-Fermi function $\chi(x)$.

(b) Find the ground state energy and wave function of a system 3 non-interacting identical particles, that are confined to a one-dimensional, infinite well when the particles are (i) bosons and (ii) spin $\frac{1}{2}$ fermions.

4+3+3

Group-B

(*Statistical Mechanics I*)

[Marks : 20]

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 2×5

(a) Let x_i be either p_i or q_i (i = 1, 2, ... 3N) and H is the Hamiltonian then what is the value of $\left\langle x_i \frac{\partial H}{\partial x_j} \right\rangle$ and $\left\langle \sum_{i=1}^{3N} q_i p_i \right\rangle$.

(b) Show that the rotational level with the highest population is given by $J_{max.} = \sqrt{\frac{IK_BT}{\hbar}} - \frac{1}{2}$.

(c) For imperfect gas, the Hamiltonian of the system is $H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i<j} v_{ij}$ and the intermolecular potential v(r) has the form $f(r) = e^{-\beta v(r)} - 1$.

Plot variation of v(r) and f(r) with r.

(d) The wave function ψ of an isolated system is given by

$$\psi = \sum_n C_n(t)\, \phi_n ,$$

where $\{\phi_n\}$ is the complete orthonormal set of stationary wave functions.
Write down postulate of equal a priori probability and random phases in terms of C_n.

(e) Consider a system of non-interacting particles in d dimensions obeying the dispersion relation $\varepsilon = Ak^s$, where ε is the energy, k is the wavevector, 's' is an integer and A is a constant. Find the density of states.

(f) The entropy of an ideal paramagnet in a magnetic field is given by $S = S_0 - CU^2$ where U is the energy of the spin system and C is a constant.
Plot U as function of temperature for $C > 0$.

(g) Prove that pure state remains pure always.

2. (a) Consider a spin-1 particle with Hamiltonian

$$\hat{h} = -\mu_0 H\sigma + \Delta\left(1 - \sigma^2\right)$$

where $\sigma = -1, 0, +1$ and Δ represents the vacancy formation energy. H is the applied magnetic field. Calculate (i) vacancy number (ii) magnetization (iii) weak field magnetic susceptibility.

What happens if $T \ll \Delta/K_B$ and $T \gg \Delta/K_B$?

2+2+2

(b) if for N distinguishable particles energy $\varepsilon = pc$ where p is the momentum.
Calculate grand-canonical partition function, and hence prove that

$$\mu = KT \ln\left[\frac{h^3c^3P}{8\pi(KT)^4}\right]$$

where P is the pressure. 4

3. (a) Deduce the equation of state of ideal Bose and Fermi gas if $\varepsilon = Ap^s$ taking into account non-relativistic and relativistic gas. 3+3

(b) Given N identical non-interacting magnetic ions of spin $\frac{1}{2}$, magnetic moment μ_0 in a crystal at absolute temperature T in a magnetic field H. Show that fluctuation of magnetic moment

$$\Delta M = \sqrt{N}\,\frac{\mu_0}{\cosh\left(\frac{\mu_0 H}{kT}\right)}.$$ 4
