2016

M.Sc. 2nd Seme. Examination

PHYSICS

PAPER—PHS-201

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer-scripts for Group-A & Group-B.

Group—A

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 5×2

(a) Show that if the Hamiltonian is invarient under a unitary transformation, the generator of the transformation is conserved.

(Turn Over)

(b) Prove that $e^{i\vec{\sigma}.\hat{n}\theta} = \cos\theta + i\,\vec{\sigma}.\hat{n}\sin\theta$.

(c) The operator $S_r = \frac{\vec{S}.\vec{r}}{r}$ is the component of the electron spin in the direction of vector $\vec{r}$. Then prove that $\left[\vec{S}, Sr\right] = i\hbar\frac{\vec{r}\times\vec{s}}{r}$.

(d) Show that $\gamma_\mu^+ = \gamma^\circ\gamma_\mu\gamma^\circ$.

(e) Prove that $\left(\vec{\alpha}.\vec{A}\right)\left(\vec{\alpha}.\vec{B}\right) = 4\vec{A}.\vec{B}$.

where $\vec{\alpha}$ is the Dirac α-matrices.

(f) Show that for Dirac perticle,

$$\sum_{r=1}^{2} u_r(p)\,\overline{u_r}(p) = \frac{\not{p} + m}{2m}$$

(g) Find the C.G. Coefficients for $J_1 = \frac{1}{2}$, $J_2 = \frac{1}{2}$.

(h) From relativistic energy relation obtain Klein-Gordon equation.

2. (a) An unperturbed two level system has energy eigen values E_1 & E_2 and eigen, functions $\begin{pmatrix}1\\0\end{pmatrix}$ & $\begin{pmatrix}0\\1\end{pmatrix}$. When perturbed, its Hamiltonism is represented by $\begin{pmatrix}E_1 & A\\A^* & E_2\end{pmatrix}$.

Find the 1st, 2nd order correction to E_1 and 1st order correction to eigen function $\begin{pmatrix}1\\0\end{pmatrix}$. 3

(b) Calculate the reflection and transmission coefficients of a Klein-Gorden particle of charge q and mass m incident on a potential barrier with energy E.

Given $A^0 = 0$ for $z < 0$

$= U_0$ for $z > 0$

where U_0 is a positive constant.

Discuss the case for

(i) $E > m + qU_0$

(ii) $E < -m + qU_0$

What is Klein paradox ? 3+3+1

3. (a) An electron is described by the following angular wave function

$$u(\theta, \phi) = \frac{1}{4}\sqrt{\frac{15}{\pi}} \sin^2\theta \cos 2\theta$$

$$\left(\text{Given } Y_{2,\pm 2}(\theta,\phi) = \sqrt{\frac{15}{32\pi}} \sin^2\theta \exp(\pm 2i\phi)\right)$$

Find the eigen value of $\hat{L}^2$ for this wave function $u(\theta, \phi)$. 2

(b) A particle of mass m is trapped in a potential well which has the form $V = \frac{1}{2}mw^2x^2$. Use the variational method with the normalized trial function $\frac{1}{\sqrt{a}}\cos\left(\frac{\pi x}{2a}\right)$ in the limit $-a < x < a$, to find the best value of a. 3

(c) Find the time dependence of the position operator

$$\vec{r}_{H_D}(t) = e^{iH_D t}\,\vec{r}\,e^{-iH_D t}$$

for a free Dirac particle. Show that motion of the particle is a superposition of classical uniform and rapid oscillatory motions. 5

Group—B

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 2×5

(a) Find the Fourier-transform of a function $\theta(x)$ where $\theta(x)$ is the unit step function.

(b) Find the Laplace-transform of the function

$$f(t) = \frac{1}{t}\left(e^{-\alpha t} - e^{-\beta t}\right).$$

(c) Write down bilinear formula for Green's function.

(d) Solve : $\left(\frac{\partial u}{\partial x}\right)^2 - \left(\frac{\partial u}{\partial y}\right)^2 = x - y.$

(e) Solve the integral equation

$$\int_0^x \left(1 - x^2 + \xi^2\right) y(\xi)\, d\xi = \frac{x^2}{2} \text{ by}$$

reducing it to Volterra's equation of second type.

(f) If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 3 & 2 & 4 \end{pmatrix}$

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$$

Then show that $f^{-1} o\, g^{-1} \neq g^{-1} o\, f^{-1}$.

(g) If H is a subgroup of G then show that $H^{-1} = H$.

(h) Show that $x' = ax + b$, $a \neq 0$ form a Lie group. Find the generators.

2. (a) Solve the integral equation

$$y(x) = \phi(x) + \lambda \int_0^x e^{x-\xi}\, y(\xi)\, d\xi$$

(Take $\lambda = 2$, $\phi(x) = \sin x$) 4

(b) Find the Green's function for the differential equation

$$\frac{d^2\phi}{dx^2} - \alpha^2\phi = 0,\ 0 \le x \le a$$

with boundary conditions

$$\phi(0) = 0 \text{ and } \frac{1}{\phi}\frac{d\phi}{dx}\bigg|_{x=a} = -\alpha$$ 4

(c) Evaluate $\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$

(By the method of Laplace transform). 2

3. (a) Character table for O_h :

O_h	E	$6C_4$	$3C_2$	$6S_4$	$8C_3$	$8S_6$	$3\sigma_h$	i	$6\sigma_d$	$6C_2'$
T_{1g}	3	1	−1	1	0	0	−1	3	−1	−1
T_{2g}	3	−1	−1	−1	0	0	−1	3	1	1
E_g	2	0	2	0	−1	−1	2	2	0	0
T_{Ag1}	1	1	1	1	1	1	1	1	1	1
G	9	1	1	1	0	0	1	9	1	1

For a group $T_{2g} \otimes T_{2g} = G$ in reducible representation the character is given above.

Find the number of irreducible representation in G and show that

$$G = A_{g1} + E_g + T_{1g} + T_{2g}. \qquad 5$$

(b) Solve $\nabla^2 \phi\left(\vec{r}\right) = -\frac{\rho\left(\vec{r}\right)}{\varepsilon_0}$ by the method of Fourier transform. 5