2016

M.Sc. 1st Semester Examination

PHYSICS

PAPER—PHS-102

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer-scripts for Group-A & Group-B

(Quantum Mechanics–I)

Group—A

Answer Q. No. *1* and any *one* from the rest.

1. Answer any five bits : 2×5

(a) Find the constant B which makes e^{-ax^2} an eigen function of the operator, $\left(\frac{d^2}{dx^2} - Bx^2\right)$, where a is a constant.

What is the corresponding eigenvalue ?

(b) Let $\hat{N} = \hat{b}^{+}\hat{b}$ where the operator b satisfies the relation $\left\{\hat{b}, b^{+}\right\} = 1$ and $\hat{b}^{2} = \left(\hat{b}^{+}\right)^{2} = 0$. Find the eigenvalues of $\hat{N}$.

(c) $|\phi_1\rangle$ and $|\phi_2\rangle$ be two orthonormal state vectors and let, $A = |\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|$. Is it a projection operator ?

(d) Evaluate $\left[\frac{\partial}{\partial x}, F(x)\right]$.

(e) Find the momentum space ware function $\phi(p)$ for the plane wave $\psi(x) = Ae^{\frac{ip_0x}{\hbar}}$.

(f) Show that $\frac{-i\hbar}{\sin\theta}\frac{\partial}{\partial\theta}$ is a self-adjoint operator in the spherical polar co-ordinate system (r, θ, ϕ).

(g) Electrons of energy 10eV are incident on a potential step of height 13.8 ev. Find the distance in which the probability density of finding the particle decreases to a factor of 0.01 as it penetrates into the classically forbidden region.

(c) A particle is confined to the region $x \geq 0$ by a potential which increases linearly as $V(x) = V_0 x$. Show that average position of the particle at temperature T is

$$\langle x \rangle = \frac{K_B T}{V_0}.$$ 4+4+2

3. (a) If the ground state wave function for the hydrogen atom is $\psi_0 = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$.

Find the expectation value of K.E of the electron in the 1S state.

(b) Given that $\hat{p}_r = -i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)$, show that the uncertainty Δp_r in the ground state is $\frac{\hbar}{a_0}$.

$$\left[\psi_0(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}\right]$$

(c) Consider the operator $\frac{d}{dr}$ for a spherically symmetric system, and a set of eigenfunction e^{-ar}, a being real and positive. Is the operator necessarily Hermitian ?

4+4+2

(h) A particle of mass 'm' is moving under a potential of the form

$$V(x) = \frac{1}{2}mw^2x^2 \quad x > 0$$
$$= \infty \quad x \leq 0$$

The wavefunction of the particle is

$$\psi(x) = \frac{1}{\sqrt{5}}\psi_0 + \frac{2}{\sqrt{5}}\psi_1$$

where ψ_0 and ψ_1 are the eigen functions of ground state and first excited state respectively. Find the expectation value of energy.

2. (a) A particle is initially in the ground state in 1–D harmonic oscillator potential $V(x) = \frac{1}{2}kx^2$. If the spring is suddenly doubled, prove that the probability of finding the particle in the ground state of new potential is

$$\frac{2^{5/4}}{1+\sqrt{2}}.$$

(b) A particle of mass m is the one-dimensional short range potential $V(x) = -V_0\,\delta(x).(V_0 > 0)$. Find the energy of the system.

Group—B

Answer Q. No. *1* and *2* any *one* from the rest.

1. Answer any *two* bits : 2×2

(a) What is Miller Bravis Indices ? Calculate the angle between [111] and [001] directions in a cubic crystal ? 1+1

(b) Draw the stereogram corresponding to rotation inversion axis such as $\bar{1}$, $\bar{2}$, $\bar{3}$, $\bar{4}$, $\bar{6}$. 2

(c) Find the structure factor of a crystal containing 2_1 screw parellal of b axis. 2

2. Answer any *two* bits : 2×3

(a) Describe Ewald constructions to show diffraction of Xrays by a crystal. Also prove equivalence between Bragg's law and Laue equation. 2+1

(b) Clearly distinguish between harmonic and anharmonic vibration ? What is umklapp process ? 2+1

(c) Prove that effective number of free electron in a solid is maximum when the outermost band is half filled. What is meant by negative effective mass ? 2+1

3. (a) Describe in details Debye Waller effect and hence find Debye Waller factor.

   (b) Derive the vibrational modes of a diatomic linear chain of atom. 5+5

4. (a) What is the physical origin of energy gap in a solid ?

   (b) Clearly distinguish between single crystal and polycrystal.

   (c) Describe in details powder Diffraction technique. 6+2+2
