2016

M.Sc. 4th Seme. Examination

PHYSICS

PAPER—PHS-401

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer-scripts for Group-A & Group-B.

Group-A

[Marks : 20]

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 5×2

(a) Why two types of vectors exist in general coordinate systems while there is only one type of vector in orthogonal coordinate system ?

(b) If the surface of a sphere of fixed radius 'a' is given by $ds^2 = a^2\, d\theta^2 + a^2 \sin^2\theta\, d\phi^2$.

Evaluate $\begin{Bmatrix} 2 \\ 22 \end{Bmatrix}$ and $\begin{Bmatrix} 2 \\ 21 \end{Bmatrix}$.

(c) Calculate the number of independent components of Riemann-Christoffel tensor.

(d) State and explain the principle of equivalence.

(e) Write down the expression for Minkowski's 4-force. Hence obtain its classical limit.

(f) Explain the phenomenon of length contraction using Minkowski's 4-Dimensional space-time representation.

(g) Prove the relation,

$$\Gamma_{\alpha,\,\mu\nu} = g_{\alpha\rho}\, \Gamma^{\rho}{}_{\mu\nu}$$

(The symbols have their standard meanings.)

(h) How would you come to the conclusion that pulsars are not white Dwarfs but are rotating Neutron Stars ?

2. (a) Assuming an equation of state of the type $P = k\rho^{\gamma}$ for a star, derive the equation $\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{dy}{dx}\right) + y^n = 0$, where y is called Lane-Emden function, n is the polytropic index $= \frac{1}{\gamma - 1}$ and $\gamma = \frac{C_p}{C_v}$, k is a constant and other symbols have their usual meanings.

From a solution of y show that for $n \geq 5$, no real star formation is possible. 5

(b) If

$$ds^2 = dt^2 - \frac{a^2(t)}{1 - kr^2}dr^2 - a^2(t)r^2 d\theta^2 - a^2(t)r^2 \sin^2\theta \, d\phi^2,$$

show that component of the Ricci tensor

$R_{00} = -3\frac{\ddot{a}}{a}$. 5

3. (a) A neutron star is assumed as sphere which consists entirely of neutrons which form a non-relativistic degenerate Fermi gas. If the mass of the star is 10^{30} kg, calculate its radius. 5

(b) It is know that 50% to 80% of mass of a typical star is made up of hydrogen. However, only a small fraction of stars shows Balmer lines in their spectra. Explain. 2

(c) Discuss 'Chandrasekhar Mass limit' for a white dwarf. 3

Group-B

[Marks : 20]

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *five* bits : 5×2

(a) Show that zero point energy of a Debye Solid is $\frac{9}{8}Nk_B\Theta_D$ where Θ_D is the Debye temperature.

(b) For a linear chain of (N+1) atoms, each of mass m, with harmonic forces acting between nearest neighbours dispersion relation

$$\varepsilon(k) = \hbar w_{max} \sin \frac{ka}{2} \text{ where length of the chain } L = Na \;;$$

a = lattice constant.

Prove that density of states $g(w) = \dfrac{2N}{\pi\sqrt{w_{max}^2 - w^2}}$.

(c) Draw the variation of chemical potential μ with temperature T of ideal Fermi gas and Bose gas.

(d) Calculate density of radiation in two dimension.

(e) Define spin-spin correlation function and how is it related to magnetic susceptibility ?

(f) State the principle of Lee-Yang phase transition.

(g) Distinguish between He-I and He-II in the light of two fluid model.

(h) Show that Bulk modulus of an electron gas at 0 K is $\frac{5}{3}P$.

2. (a) Consider a gas of N spin-zero bosons in a d-dimensional container of volume V, with a dispersion relation

$$\varepsilon_{\vec{p}} = \alpha|\vec{p}|^{s}$$

where the constant α and the index S are both positive.

(i) Find expression for the mean number of particles per unit volume in the ground state and the mean total number of particles in the excited states, in terms of the temperature T and the fugacity $z = e^{\mu\beta}$.

(ii) Find the conditions on s and d for which Bose-Einstein condensation takes place.

(iii) Find the equation of state for this gas.

2+1+1+1

(b) For a 2nd order-phase transition Gibb's free energy is given by

$$G(T, m) = G_0(T) + a(T)m^2 + b(T)m^4 + \cdots\cdots$$

where m is the order parameter. Obtain the possible values of m for stable phase. Prove that entropy is continuous at T_c with the help of G-L theory. If magnetic field is applied, how it affects the symmetry of the system ?

2+2+1

3. (a) A lattice of (N + 1) sites has spins $S_i = \pm 1$ at each site, all of which are acted on by a magnetic field. There are interactions of equal strength between one of the spins, S_0 and each of the other.

$$\hat{H} = -h\sum_{i=0}^{N} S_i - J_{ex}\sum_{i=1}^{N} S_i S_0$$

(i) Find the canonical partition function Q(T, N), the average energy $\langle E \rangle$ and for $i \neq 0$, the statistical averages $\langle S_i \rangle$ and $\langle S_0 S_i \rangle$. Compute the limits of these averages when $h \to 0$ with $J_{ex} \neq 0$ and $J_{ex} \to 0$ with $h \neq 0$.

(c) (i) Write down the expression for free energy of Fermi gas under quantization by magnetic field.

(ii) If $\varepsilon(p_z, j) = \frac{p_z^2}{2m} + \frac{cB\hbar}{mc}\left(j + \frac{1}{2}\right)$ for an electron in a uniform magnetic field $\vec{B} = B\hat{z}$. The energy levels are degenerate, the number of states for a given level being $g = 2V^{2/3}eB / 2\pi\hbar c$.

Write down the grand canonical partition function for a gas of electrons in a magnetic field. Also show that in the high temperature limit, the zero-field diamagnetic susceptibility varies as $-\frac{1}{T}$.

Show that for weakly degenerate Bose gas

$$C_V = \frac{3}{2} N k_B \left(1 + \frac{A}{2^{7/2}}\right)$$

where $A = e^{\mu/k_B T}$. 3+2
