2016

M.Sc. 1st Semester Examination

PHYSICS

PAPER—PHS-101

Full Marks : 40

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answer-scripts for Group-A & Group-B

(Methods of Mathamatical Physics)

Group—A

Answer Q. No. *1* and any *one* from the rest.

1. Answer any five bits : 2×5

(a) Show that $(1 + x + iy)^4 (7 - x - iy)^3$ is an analytic function of the complex variable $z = x + iy$ in the domain $|z| < 2$.

(b) Show that $\text{erf}(-x) = -\text{erf}(x)$.

(c) Show that $(1 + i)$ and $(1 - i)$ be the eigen values of $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$.

(d) State and prove Schwart inequality.

(e) Evaluate associated Legendre polynomial $P_3^{\ 1}$.

(f) $r^2 \dfrac{d^2R}{dr^2} + 2r\dfrac{dR}{dr} + \left[k^2r^2 - n(n+1)\right] R = 0$

Reduce this equation into Bessel's equation of suitable order.

(g) Write down the integral form of $H_n^{(x)}$.

(h) Show that $H'_{2n}(0) = 0$.

2. (a) If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ then show that $e^A = I + \dfrac{A}{3}\left(e^3 - 1\right)$.

(b) A 2×2 matrix 'A' has eigenvalues $e^{\frac{i\pi}{5}}$ and $e^{\frac{i\pi}{6}}$. Find the smallest value of 'n' such that $A^n = I$.

(c) Evaluate

$$\int_0^{\infty} \frac{\log(1+x^2)}{(1+x^2)} dx \text{ by the method of residue.}$$

3+3+4

3. (a) Prove that

$$\int_{-1}^{+1}(1-x^2)\left\{\frac{d}{dx}P_n(x)\right\}^2 dx = \frac{2n(n+1)}{(2n+1)}.$$

(b) Show that

$$\int_{-\infty}^{+\infty} e^{-Ax^2+Bx} dx = \sqrt{\frac{\pi}{A}}\, e^{\frac{B^2}{4A}}$$

(c) Show that

$$Y_n^{-m}(\theta, \phi) = (-1)^m Y_n^{m*}(\theta, \phi).$$

3+4+3

Group—B

Answer Q. No. *1* and any *one* from the rest.

1. Answer any *four* of the followings : $4\times2\frac{1}{2}$

(a) Show that for a single particle with constant mass the equation of motion implies the following differential equation for the kinetic energy :

$$\frac{dT}{dt} = \vec{F}.\vec{v}$$

while if the mass varies with time the corresponding equation is

$$\frac{d}{dt}(mT) = \vec{F}.\vec{p}$$

(b) The Lagrangian of a problem is

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + V(r).$$

Identify the cyclic coordinate and the corresponding law for the problem.

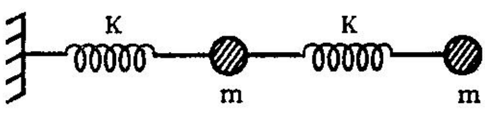
(c) Show that the transformation $Q = p + iaq$ and $P = \frac{p - iaq}{2ia}$ is canonical and find out the generating function.

(d) Derive Hamilton's canonical equation in terms of Poisson bracket.

(e) A bead of mass m is constrained to move under gravity along a planar rigid wire that has a parabolic shape $y = x^2/l$, where x and y are the horizontal and the vertical coordinates respectively. Derive the Lagrangian of the system and the equation of motion.

(f) Consider a particle of mass m moving in one dimension under a force with the potential $U(x) = K(2x^3 - 5x^2 + 4x)$, where $K > 0$. Show that the point $x = 1$ corresponds to a stable equilibrium position for the particle.

2. (a) Two identical beads each of mass m each can move without friction along a horizontal wire and are connected to a fixed wall with two identical springs of spring constant K as shown in the figure below :



Find the Lagrangian for this system and derive from it the equation of motion. Find the eigen frequencies of small oscillations.

(b) Using Hamilton Jacobi theory, derive the equation of motion of a particle, which is falling under gravity.

2+3+5

3. What is action-angle variable ? Find out the frequency of a linear harmonic oscillator using action-angle variable method. Starting from the time-dependent Schrödinger equation obtain the Hamilton-Jacobi equation. 2+5+3

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