2016

MCA 1st Semester Examination

DISCRETE MATHEMATICS

PAPER—MCA-102

Full Marks : 100

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer any *five* questions.

1. (a) Let A, B, C be subsets of a universal set S. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ 7

(b) Prove that

$$\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = \left(a^2 + b^2 + c^2 + d^2\right)^2$$ 7

2. (a) Prove that

$$\begin{vmatrix} a+1 & a & a & a \\ a & a+2 & a & a \\ a & a & a+3 & a \\ a & a & a & a+4 \end{vmatrix} = 24\left(1+\frac{a}{1}+\frac{a}{2}+\frac{a}{3}+\frac{a}{4}\right)$$

7

(b) If $A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$, show that $A^2 - 10A + 16I_3 = 0$.

Hence obtain A^{-1}. 7

3. (a) Prove that the matrix $\frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{pmatrix}$ is orthogonal. Utilise this to solve the equations.

$$x - 2y + 2z = 2$$
$$2x - y - 2z = 1$$
$$2x + 2y + z = 7$$

7

(b) (i) Define limit of a sequence. 2

(ii) Test the series :

$$1 + \frac{x}{1!} + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \ldots, \; x > 0$$

5

4. (a) (i) Prove that the sum of the degrees of all vertices in a graph is twice the number of edges in the graph. 3

(ii) Define Walk and Regular graph. 2+2

(b) (i) Draw, if possible, a simple graph with five vertices having degrees 2, 3, 3, 3, 3. 5

(ii) Define eccentricity of a vertex. 2

5. (a) If n be a positive integer then show by using the principle of mathematical induction that $10^n + 3.4^{n+2} + 5$ is divisible by 9. 7

(b) Let $f : R \rightarrow R$ (the set of all Real numbers) be defined by $f(x) = 3x + 1, x \in R$.
Examine if f is (i) injective, (ii) surjective. 7

6. (a) (i) Find f.g and f^{-1} where

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 5 & 6 & 1 \end{pmatrix} \text{ and } g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 4 & 5 & 3 & 2 \end{pmatrix}$$

2+2

(ii) Find the images of the elements 3 and 4 if

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & & & 3 \end{pmatrix}$ be an odd permutation. 3

(b) Solve the following equation using generating function $a_n - 5a_{n-1} + 6a_{n-2} = 2^n + n$, $n \geq 2$ with the boundary conditions $a_0 = 1$, $a_1 = 1$. 7

7. (a) Express the following expression as a function in conjunction normal form :

$$xyz + xy'z' + x'yz' + x'y'z'$$ 7

(b) (i) Define Euler line, Euler graph and Euler circuit. 4

(ii) Define Hamiltonial Circuit and Hamiltonial Path. 3

[Internal Assessment : 30]
