2016

M.Sc. 1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-103

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer Q. No. *1* is compulsory and any *three* from the rest.

1. Answer any *five* questions : 2×5

(a) When are the two integrable real-valued functions f(x) and g(x) with weight function $\rho(x) > 0$ on an interval I said to be orthogonal ?

(b) Reduce confluent hypergeometric equation from hypergeometric equation.

(Turn Over)

(c) Find the indicial equation of the Bessel equation

$$z^2 \frac{d^2w}{dz^2} + z\frac{dw}{dz} + \left(z^2 - v^2\right)w = 0$$

corresponding to its singularity.

(d) What is Bessel's formations of order n. State for what values of n the solutions are independent of Bessel's equation of order n.

(e) Let $P_n(z)$ be the Legendre polynomial of degree n and let

$$P_{m+1}(0) = -\frac{m}{m+1} P_{m-1}(0),\ m = 1, 2, \ldots$$

If $P_n(0) = -5/16$, then find the value of $\int_{-1}^{1} P_n^2(z)\, dz$.

(f) Write the hypergeometric series represented by

F(a, b, c ; z). Prove that $F(1, b, b ; z) = \frac{1}{1-z}$.

(g) Show that $P_n(-z) = (-1)^n P_n(z)$.

2. (a) Prove that all the eigen values of a regular Sturm-Liouville System with non-negative weight function are real. 6

(b) Show that $J_0^2(z) + 2\sum_{n=1}^{\infty} J_n^2(z) = 1$ and prove that for real z, $|J_0(z)| \le 1$, and $|J_n(z)| < \frac{1}{\sqrt{2}}$, for all n > 1. 4

3. (a) Prove that if f(z) is continuous and has continuous derivatives in [−1, 1] then f(z) has unique Legendre series expansion is given by

$$f(z) = \sum_{n=0}^{\infty} C_n P_n(z)$$

where P_n's are Legendre polynomials and

$$C_n = \frac{2n+2}{2} \int_{-1}^{1} f(z)\, P_n(z)\, dz \text{, for } n = 1, 2, 3, \ldots$$ 6

(b) Deduce the integral formula for hypergeometric function. 4

4. (a) Examine that whether the following matrix

$$\varphi(t) = \begin{pmatrix} 2e^t & e^{2t} & 0 \\ 2e^t & 2e^{2t} & 3e^{5t} \\ e^t & e^{2t} & e^{5t} \end{pmatrix}$$

is a fundamental matrix of the linear system

$$\frac{dx}{dt} = \begin{pmatrix} 0 & -2 & 6 \\ -2 & 9 & -12 \\ -1 & 2 & -1 \end{pmatrix} x$$

where $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ or not. Justify your answer. 5

(b) Find the characteristic values and characteristic functions of Sturm-Liouville problem

$$\frac{d}{dx}\left[x\frac{dy}{dx}\right]+\frac{\lambda}{x}y=0$$

with boundary conditions, $y(1) = 0$, $y^1(e^\pi) = 0$. 5

5. (a) Find the general solution of the equation

$$2z(1-z)\frac{d^2w}{dz^2}+\frac{dw}{dz}+4w(z)=0$$

by Frobenius method about $z = 0$ and show that the equation has a solution which is polynomial in z.

6

(b) Show that $J_0(kz)$, where k is a constant, satisfies the differential equation

$$z^2\frac{d^2w}{dz^2}+\frac{dw}{dz}+k^2z\,w=0.$$ 4

6. (a) Find the Green's function of the following differential equation

$$\frac{d^3y}{dx^3}=0,\ y(0)=y(1)=0$$

and $y^1(0) = y^1(1)$. 6

(b) Find the generating function for Bessel function of integral order. 4

(Internal Assessment : 10 Marks)
