2016

M.Sc. 4th Seme. Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-401

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*Functional Analysis*)

Answer Q. No. *1* and any *four* from *Q. No. 2* to *Q. No. 6.*

Symbols have their usual meanings.

1. Answer any *four* questions : 4×2

(a) Let F and G be subspaces of a Hilbert Space H. Show that $(F+G)^{\perp} = F^{\perp} \cap G^{\perp}$.

(b) For $y \in l^{\infty}$, define $f_y(x) = \sum_{j=1}^{\infty} x(j)\, y(j)$, $x \in l^1$. Show that $\|f_y\| = \|y\|_{\infty}$.

(Turn Over)

(c) Check whether $C^1[0,1]$ with the supremum norm is a Banach space.

(d) If in an inner product space X, $x_n \to x$, $y_n \to y$, then show that $\langle x_n, y_n \rangle \to \langle x, y \rangle$.

(e) Define the following with examples : Unitary operators, Strictly positive operators.

(f) Let X be a complete normed linear space & $u : X \longrightarrow \mathbb{R}$ be a real linear functional. If $f(x) = u(x) - iu(ix)$, $x \in X$, then, for α, $\beta \in \mathbb{R}$, show that $f((\alpha + i\beta)x) = (\alpha + i\beta)f(x)$, $x \in X$.

2. (a) Let X be a normed linear space and X_0 be a closed subspace of X. If $x \in X$, then show that $x \notin X_0$ if and only if there exists a non-zero linear functional $\phi \in X^*$ such that $\phi(x) \neq 0$ and $\phi(y) = 0$ for all $y \in X_0$.
(X^* denotes the dual of X).

(b) Let L be a closed linear subspace of NLS X, prove that $\|X + L\| = \inf \{\|x + y\| : y \in L\}$ for all $x \in X$ is a norm function on the quotient space X / L. 5+3

3. (a) State the open Mapping theorem. Let $X = C^1[0,1]$ equipped with the norm $\|x\| = \|x\|_\infty + \|x'\|_\infty$ and $Y = C[0, 1]$ equipped with the supremum norm $\|x\|_\infty$. Check whether the linear operator $F:X \to Y$ defined by $F(x) = x$ is contiuous. Is F an open map ?

(b) Let X and Y be Banach spaces over same scalars and $T:X \to Y$ be a linear operator. Define the graph G (T) of T and prove that G (T) is a subspace of $X \times X$. 5+3

4. (a) Let X and Y be two Banach spaces and $T \in B(X,Y)$ be bijective . Prove that there exists $S \in B(Y,X)$ such that $ST = I_X$ and $TS = I_Y$.

(b) Assume that $\{U_\alpha\}_{\alpha \in I}$ is an orthonormal set in the inner product space X and $x \in X$. If $E_x = \{U_\alpha : \langle x, U_\alpha \rangle \neq 0\}$, then show that E_x is a countable set. 4+4

5. (a) Let H be a Hilbert space and Y be an inner product space. If $T \in BL(H,Y)$. Then define the adjoint of T ie, T^*. Also, give an example to show that if H is simply an inner product space, then $T \in BL(H,Y)$ may not have an adjoint.

 (Turn Over)

(b) Let $T \in BL(H)$ be normal, where H is a Hilbert space. Show that if $Tx = \alpha x$ for some $x \in H$, then $T^*x = \bar{\alpha}x$ where α is a scalar. 1+4+3

6. (a) Let H be a complex Hilbert space and B be the space of all bounded linear operators $T: H \to H$. Prove that the following statements are equivalent

(i) $T^*T = I$ (Identity operator),

(ii) $(T_x, T_y) = (x, y)$ for all $x, y \in H$,

(iii) $\|T_x\| = \|x\|$.. for all $x \in H$.

(b) Let $T_n (n \geq 1)$, $T \in BL(X, Y)$ where X and Y are normed linear spaces. Then, define the weak and strong convergence of the sequence of operators $\{T_n\}_{n \geq 1}$ to T.

(c) Let X be a finite dimensional inner product space and $\{x_n\}_{n \geq 1}$ be a sequence in X. Then show that $x_n \longrightarrow x$ if and only if $x_n \xrightarrow{\omega} x$. 4+2+2

[*Internal Assesment : 10 Marks*]