2016

M.Sc. 1st Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-102

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer Q. No. *1* and any *four* from the rest.

1. Answer any *four* questions : 4×2

(a) Find the value of $\int_C \operatorname{Log}(z+4)dz$

where $C : 9x^2 + 4y^2 = 36$.

(b) Find $\operatorname{Res}_{z=1}\left(\frac{\operatorname{Cos}\pi z}{1-z^{976}}\right)$.

(c) State Cauchy-Riemann equations.

(Turn Over)

(d) Sketch $S = \left\{ z : \left| \frac{z+1}{z-1} \right| < 1 \right\}$ and decide whether it is domain.

(e) Prove that f(z) = Real(z) is nowhere differentiable.

(f) Find the critical points of $W = \frac{\alpha z + \beta}{\gamma z + \delta}$, $\alpha\delta - \beta\gamma \neq 0$.

2. (a) Given f(z) to be analytic, show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4|f'(z)|^2.$$

(b) Find the general and principle value of $\left(1 + \sqrt{3}i\right)^{5i}$. 3

3. (a) Evaluate $\int_C e^{\bar{z}}\, dz$, where C is the polygonal path consisting of the line segments from z = 0 to z = 2 and from z = 2 to $z = 1 + iT_1$.

(b) Show that

$$|\sinh(\operatorname{Im} z)| \leq |\sin z| \leq |\cosh(\operatorname{Im} z)|$$ 5+3

4. (a) Without evaluating, find an upper bound of

$$\int_C \frac{e^{2z} - \frac{\sqrt{3}}{2}\bar{z}}{z^2 + 2}\, dz,$$

where C is the arc of $|z| = \sqrt{3}$ from $|z| = -\sqrt{3}$ to $z = -i\sqrt{3}$, taking anti clockwise direction.

(b) Show that a function analytic everywhere including the point at infinity is a constant function. 5+3

5. (a) Let f(z) be analytic at z_0, and consider $g(z) = \dfrac{f(z)}{z - z_0}$.

Show that

(i) if $f(z_0) \neq 0$, then z_0 is a simple pole of g(z) and

$\underset{z = z_0}{\text{Res}}\ g(z) = f(z_0)$, and

(ii) if $f(z_0) = 0$, then z_0 is a removal singularity of g(z) and

$\underset{z = z_0}{\text{Res}}\ g(z) = 0$.

(b) Find the square roots of 2i. 5+3

6. (a) Evaluate $\displaystyle\int_0^{\infty} \frac{x^2}{\left(x^2+1\right)\left(x^2+4\right)}\,dx$, using the method of residue only.

(b) Show that

$\log(i^2) \neq 2 \log i$ when $\log z = \ln r + i\theta$

$\left(r > 0,\ 3\frac{\pi}{4} < \theta < \frac{11\pi}{4}\right)$. 5+3

7. (a) Using the method of residue,

evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$.

(b) Find all the Mõbius Transformation which transform the half plane $I(z) \geq 0$ onto the unit circular disc $|w| \leq 1$. 5+3

(Internal Assessment : 10 Marks)
