2016

M.Sc.

3rd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-303

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

[Dynamical Oceanology and Meteorology / Operational Research]

(Dynamical Oceanology and Meteorology)

Answer Q. No. *1* and any *four* from the rest.

1. Answer any *four* questions : 4×2

   (a) Find a relation between mixing ratio and specific humidity. 2

   (b) Deduce the dry adiabatic lapse rate. 2

   (c) Show that the potential temperature of an air parcel is invariant. 2

(Turn Over)

(d) Write the properties of Reynolds operator of averaging.

(e) For typical horizontal and vertical length scales of 1000 km and 1 km, respectively, and horizontal speeds of 0.1m/s, estimate a typical vertical speed.

(f) Define all the eddy viscosities with the help of their respective turbulent stress.

2. (a) Deduce the equation of state for moist air in the atmosphere. 5

(b) Derive the geostrophic wind equation in the atmosphere. 3

3. (a) Discuss the different cases of pressure changes in the atmosphere with respect to altitude. 5

(b) Explain the convergence and divergence in the atmosphere. 3

4. (a) Derive the expression of the pressure gradient force in the atmosphere. 5

(b) Discuss different types of fronts in the atmosphere. 3

5. (a) Write the transformation in vector form ideal axes fixed in space to axes fixed to earth, then using this transformation derive the equation of motion in axes fixed to earth.

(b) For the situation as Q. No. 1(e), scale the x-momentum equation and hence derive the corresponding approximate equation. 4+4

6. Derive the Reynolds equation for y-momentum equation.

8

7. (a) Write the necessary assumptions for inertial motion and then write the inertial equation.

(b) Find the solution of the above equation and discuss its physical behaviour.

(c) What is the Rossby number and period of revolution for the above motion. 3+3+2

8. (a) Show that density lapse rate satisfies

$$-\frac{g}{\rho}\frac{d\rho}{dX} = N^2 + \frac{g^2}{C_s^2}$$

where $C_s^2 = \frac{pC_p}{\rho C_N}$, C_s is the speed of sound in air.

Comment on the implication of static stability if density decreases very slowly with height.

(b) Calculate the period of Oscillation of an air parcel given that $\frac{dT}{dz} = -6.5°k / km$ and $T = 270K$.

(Internal Assessment : 10 Marks)

(Operational Research)

Answer Q. No. *1* and any *two* from the rest.

1. Answer any *four* questions : 4×2

(a) State the sufficient conditions for multivariate optimization. 2

(b) Discuss the effect of deletion of an existing variable in optimal table of LPP. 2

(c) What is Gomory's constraint ? 2

(d) What do you mean by the term EOQ (Economic Order Quantity) ? 2

(e) What is difference between Wolfe's method and Beale's method ?

(f) What is the main principle of dynamic programming method ?

2. (a) Describe the procedure to solve an LPP by revised simplex method. What are the advantages of it over simplex method. 6+2

(b) Consider the following LPP

$$\text{Max. } Z = 15x_1 + 45x_2$$

$$\text{Sub. to} \quad x_1 + 16x_2 \leq 240$$

$$5x_1 + 2x_2 \leq 162$$

$$x_2 \leq 50$$

$$x_1, x_2 \geq 0$$

The optimal table in simplex method is found to be

C_B	Basis	K_B	y_1	y_2	y_3	y_4	y_5
45	x_2	$\frac{173}{13}$	0	1	$5/78$	$1/78$	0
15	x_1	$\frac{352}{13}$	1	0	$-1/39$	$8/39$	0
0	x_5	$\frac{477}{13}$	0	0	$-5/78$	$1/78$	1

Find the range of the values of C_1 and C_2 for which current optimal solution remains optimal when changed one at a time. 4+4

3. (a) Explain dynamic programming technique to solve the following problem :

Maximize $Z = f_1(y_1)\, f_2(y_2) \ldots f_n(y_n)$

subject to $a_1y_1 + a_2y_2 + \ldots + a_ny_n = b$

where $y_j \geq 0$, $a_j \geq 0$ for all $j = 1, 2, \ldots, n$.

Hence, find y_1, y_2, y_3 such that $y_1y_2y_3$ is maximum and $y_1 + y_2 + y_3 = 24$, $y_j \geq 0$, $j = 1, 2, 3$. 8

(b) What is quadratic programming problem ? Derive Kuhn-Tucker conditions for quadratic programming problem. Under what conditions, the conditions Kuhn-Tucker condition will be necessary and sufficient ?

2+5+1

4. (a) Find the optimal ordering quantity for a product for which the price breaks are as follows :

Quantity	Unit cost (Rs.)
$0 < q < 500$	10.00
$500 \le q < 750$	9.25
$750 \le q$	8.75

The monthly demand for the product is 200 units, storage cost is 2% of the unit cost and the ordering cost is Rs. 100.00. 8

(b) Solve the following LPP :

$$\text{Max. } Z = x_1 + 4x_2$$
$$\text{sub. to } 2x_1 + 4x_2 \le 7$$
$$5x_1 + 3x_2 \le 15$$
$$x_1, x_2 \ge 0 \text{ and integers.}$$

8

(Internal Assessment : 10 Marks)
