2016

M.Sc.

3rd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-301

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

[Partial Differential Equations and Generalized Functions]

Answer Q. No. *1* and any *two* questions from the rest.

1. Answer any *two* of the following : 2×4

(a) Define Dirac-delta function. Find the fourier transform of the Dirac delta function.

(Turn Over)

(b) Define well-posed mathematical problem with example. Also give an example of an ill-pased problem.

(c) State the Basic existence theorem for Cauchy problem of the first order quasi-linear PDE.

2. (a) Show that the equation $u_{xx} + 6u_{xy} - 16u_{yy} = 0$ is hyperbolic. Find the canonical form of the equation. Also find the general solution $u(x, y)$. 8

(b) Define the adjoint for a second order linear partical differential operator. Hence find the adjoint of the following :

$$u_{xy} + u_x - u_y = 0.$$ 3

(c) Show that the type of a linear second order PDE in two variables is invariant under a change of co-ordinates. 5

3. (a) Solve the following using the method of separation of variables :

$$u_{tt} - c^2u_{xx} = 0,\ 0 < x < l,\ t > 0$$

$$u_x(0, t) = u_x(l, t) = 0,\ t \geq 0$$

$$u(x, 0) = f(x),\ 0 \leq x \leq l$$

$$u_t(x, 0) = g(x),\ 0 \leq x \leq l,$$

where f, g are given functions. 8

(b) Solve the Darboux problem :

$$u_{tt} - u_{xx} = 0, \ t > \max\{-x, x\}, \ t \geq 0,$$

$$u(x,t) = \begin{cases} \phi(t), & x = t, \ t \geq 0 \\ \psi(t), & x = -t, \ t \geq 0, \end{cases}$$

where ϕ, ψ c^2 ([0, ∞)) satisfies $\phi(0) = \psi(0)$. 8

4. (a) State and prove the weak minimum principle. Hence show that the Dirichlet problem for the poisson's equation has at most one solution in a bounded domain. 6

(b) Define the following with example :

Test functions, Distribution, Convergence of sequence of test function, Derivative of the distribution. 6

(c) Establish the poisson formula for the Dirichlet Problem of Laplace equation in a disk of radius a. 4

(Internal Assessment : 10 Marks)