2016

M.Sc. 2nd Seme. Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-203

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(Abstract Algebra and Linear Algebra)

Unit-I

(Abstract Algebra)

[Marks : 25]

Answer Q. No. *1* and any *two* from the rest.

1. Answer any *two* questions : 2×2

(a) Show that $Z_2 \times Z_{30}$ is isomorphic to $Z_{10} \times Z_6$.

(b) Let G be a group and $f : G \to G$ be defined by $f(a) = a^n$, $\forall\, a \in G, n \in Z^+$. Suppose f is an isomorphism. Then show that, $a^{n-1} \in Z(G)$, $\forall\, a \in G$.

(c) Define principal ideal and maximal ideal with example.

2. (a) State and prove the second Isomorphism theorem of groups. 4

(b) Show that S_3 is a solvable group. 2

(c) Consider the group $G = Z_4 \times Z_6$. Let $H = \langle(\bar{0}, \bar{2})\rangle$. What are the orders of G, H, $\frac{G}{H}$? Compute $\frac{G}{H}$. 2

3. (a) Let G be the group of matrices

$$\left\{\begin{pmatrix} 1 & 0 \\ a & b \end{pmatrix} : a \in Z_7 \ \&\ b \in Z_7 \setminus [0]\right\}$$

w.r.t. matrix multiplication. Show that G is not simple. How many subgroups of order 7 are there in a simple group of order 168 ? Justify your answer. 3+2

(b) Define the following :

Group action, stabilizers of group action, Kernel of group action, normalizers of group action. 3

4. (a) Show that every maximal ideal in a commutative ring with unity is a prime ideal. 4

(b) Define the following :

Euclidean domain, Principal ideal domain, Unique factorization domain.

Give an example to show that every principal ideal domain is not an Euclidean domain. 3+1

[*Internal Assessment —0 5*]

Unit-II

(Linear Algebra)

[Marks : 25]

Answer Q. No. *5* and any *two* from the rest.

5. Answer any *two* questions : 2×2

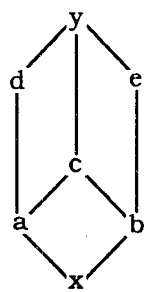
(a) Does there exists a linear transformation $T : P_1 \to P_1$ such that

$$T(1 - t) = 1, T(1 + t) = t ?$$

If yes, find it.

(b) What do you mean by annihilate? Define monic polynomial?

(c) Consider the lattice L in the following figure :



Is $L_1 = \{x, a, b, y\}$ a sublattice of L?

6. (a) Prove that a lattice L is distributive iff the equations

$a \wedge c = b \wedge c$ and $a \vee c = b \vee c$ imply $a = b$, $a, b, c \in L$.
4

(b) Let $T : R^4 \to R^3$ be the linear transformation whose matrix

relative to the basis B, B′ is $\begin{pmatrix} 1 & 2 & 1 & -1 \\ -1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 4 \end{pmatrix}$

where B is the standard basis and $B' = \{v_1, v_2, v_3\}$ with

$$v_1 = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}.$$

Find $T(e_1)$, $T(e_2)$, $T(e_3)$ and $T(e_4)$ relative to the standard basis. 4

7. (a) Define $T : P_3(R) \to M_{2\times2}(R)$ by

$$T(f) = \begin{pmatrix} f(1) & f(2) \\ f(3) & f(4) \end{pmatrix}$$

Examine whether T is invertible or not. 2

(b) Determine all possible Jordan canonical forms for a linear operator T whose characteristic polynomial is $(t-2)^5$ and minimal polynomial is $(t-2)^2$. 2+2

(c) For any $n \in N$, let P_n denotes the vector space of all polynomials with real coefficients and of degree at most n. Define $T : P_n \to P_{n+1}$ by $T(p(x)) = p'(x) - \int_0^x p(t)dt$.

Find the dimension of the null space of T ? 2

8. (a) Find the minimal polynomial of T, for each linear operator T on vector space V, where $V = P_2(R)$ and $T(f(x)) = -xf''(x) + f'(x) + 2f(x)$. 3

(b) Let T be the linear operator on $\mathbb{R}^3$ defined by

$$T(x, y, z) = (2y + z, x - 4y, 3x)$$

Find the matrix representation of T relative to the basis

$$S = \{u_1, u_2, u_3\} = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}.$$ 3

(c) Prove that a chain is a distributive lattice. 2

[***Internal Assessment —05***]