2016

M.Sc.

## 1st Semester Examination

**ELECTRONICS** 

PAPER-ELC-101

Full Marks: 50

Time: 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

## (Mathematical Methods and Numerical Analysis)

Answer Q. No. 1 and any three questions from the rest.

## 1. Answer all questions:

5×2

- (a) What are absolute and relative errors?
- (b) Show that  $E = 1 + \Delta$ where E = shift operator.

(c) Find the inverse Laplace transform of

$$\frac{S^2 - 3S + 4}{S^3}$$
.

(d) If  $w = \log z$ , find  $\frac{dw}{dz}$  & determine where w is analytic.

(e) x 1 2 3 f(x) 5 10 15

for what value of x, f(x) = 7.

- 2. (a) Deduce an expression for Newton's forward interpolation polynomial.
  - (b) The following table gives distance in nautical miles of the visible horizon for the given heights in feet above the earth surface:

x 100 150 200 250 300 350 400 y 10.63 13.03 15.04 16.81 18.42 19.9 21.27

Find the value of y when x = 218 ft.

- 3. Evaluate  $\int_{0}^{6} \frac{dx}{1+x^2}$  by using
  - (i) Trapezoidal rule;
  - (ii) Simpson's  $\frac{1}{3}$  rule;
  - (iii) Simpson's  $\frac{3}{8}$  rule;

Compare the results.

 $3 \times 3 + 1$ 

6

(a) Prove that the remainder in approximating f(x) by the interpolation polynomial using interpolating points x<sub>0</sub>, x<sub>1</sub>, x<sub>2</sub>, ..., x<sub>n</sub> is of the form

$$\frac{w(x) f^{(n+1)}(l\xi)}{(n+1)!}$$

where  $w(x) = (x - x_0) (x - x_1) (x - x_2) ... (x - x_n)$  and  $l\xi$  lies between the smallest and the longest of the numbers  $x_0, x_1, ..., x_n$ .

(b) If f(z) is analytic within and on a closed curve C and if 'a' is any point within C, then show that

$$f(a) = \frac{1}{2\pi i} \int_{a}^{b} \frac{f(z)dz}{z-a}.$$

- 5. (a) Describe the floating point representation of a real number on a computer memory location.
  - (b) Solve the initial value problem

$$\frac{dy}{dx} = x - y^2, y(0) = 1$$

to find y(0.4) by second order Runge-Kutta method using step length 0.1.

6. (a) Prove that  $\frac{d}{dx}[x^nJ_n(x)] = x^nJ_{n-1}(x)$ 

where  $J_n(x)$  is a Bessel function of first kind of order n.

(b) Suppose the random variables X and Y have joint density function

$$f(x,y) = \begin{cases} e^{-y}, & \text{if } 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional density function of X given that Y = y.

Internal Assessment — 10 Marks