2015

M.Sc. Part-I Examination

PHYSICS

PAPER—V

Full Marks : 75

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answerscripts for Gr. A & Gr. B.

Group—A

[Marks—35]

1. Answer any *three* questions of the following : 3×2

(a) Evaluate $\Delta^2\left(3e^x\right)$.

(b) Distinguish between PAM and RAM.

(c) What are (i) Computed Go To Statement (ii) Assign Go To Statement ?

(d) What is Function Subprogram ?

(e) Define absolute error and relative error.

2. Answer any *three* questions of the following : 3×3

(a) Write a Fortran program to find a real root of the equation $x^3 - x - 1 = 0$ by iteration method.

(b) Write a Fotran program to find the value of ${}^{n}C_r$ using function subprogram.

(c) Write a program to test whether a given integer is even or odd.

(d) Write a Fortran program to obtain the scalar product of two vectors $X = (x_1, x_2, \ldots, x_n)$ and

$$Y = (y_1, y_2, \ldots, y_n)$$

given by scalar product $= \sum_{i=1}^{n} x_i y_i$

(e) Write a Fortran program to solve the following equation :

$$a_1 x + b_1 y = c_1$$

$$\text{and } a_2 x + b_2 y = c_2$$

3. Answer any *four* questions : 4×5

(a) Using Newton-Raphson's method evaluate to three decimal figures, the root of the equation $e^x = 3x$ lying between 0 and 1.

(b) Apply Runge-Kutta method (fourth order), to find an approximate value of y when $x = 0{\cdot}2$

(Given that $\frac{dy}{dx} = x + y^2$ and $y = 1$ when $x = 0$).

(c) Solve by Gauss elimination method :

$$6x - y - z = 19$$

$$3x + 4y + z = 26$$

$$x + 2y + 6z = 22$$

(d) Deduce Simpson's $\frac{1}{3}$ rule for numerical integration.

(e) Describe least square method to fit the parabola $y = a + bx + cx^2$.

(f) Find the approximate value of $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$ using Simpson's rule.

Group—B

[Marks—40]

Answer Q. No. *1* and any *three* from the rest.

1. Answer any *five* bits : 5×2

(a) Show that any square matrix can be expressed as the sum of symmetric and anti-symmetric matrix.

(b) If m < n, pvove that $\frac{d^m}{dx^m} H_n(x) = \frac{2^m n!}{(n-m)!} H_{n-m}(x)$

(c) Find the residue of $f(z) = \frac{1-e^{2z}}{z^4}$ at all its poles in the finite plane.

(d) Evaluate $L^{-1}\left(\cot \frac{S}{\omega}\right)$

(e) What are the degree and the order of the differential equation

$$\frac{d^3y}{dx^3} + 2\sqrt{\frac{dy}{dx}} + x^2 y = 0 \text{ ?}$$

(f) Prove that equivalent representations have the same set of character.

(g) If a group is defined as $a * b = a + b - 1$ Find the inverse of the element 'a'.

2. (a) Using Residue theoren prove that

$$\int_0^\infty \frac{\text{Cos } mx}{x^2 + a^2} dx = \frac{\pi}{2a} e^{-ma} \qquad (m \geq 0,\ a \geq 0)$$

(c) Show that $L^{-1}\left[\ln\left(1+\frac{\omega^2}{s^2}\right)\right] = \frac{2}{t}(1 - \text{Cos}\,\omega t)$ 5+5

3. (a) Find The Fourier transform of $\delta(x-a)$.

(b) Express $2x^2 + x + 3$ in terms of Legendre's polynomial.

(c) Using Gram Schmidt procedure find the orthonormal basis for :

$$e_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\-1\\1\end{pmatrix};\ e_2 = \frac{1}{\sqrt{6}}\begin{pmatrix}2\\-1\\-1\end{pmatrix};\ e_3 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$$ 2+3+5

4. (a) Solve :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x\, \partial y} = \text{Sinx Cos } 2y$$ by Lagrange's method.

(b) The maxtrix $A = \begin{pmatrix}a & h\\h & b\end{pmatrix}$ is transformed to the diagonal form $D = T^{-1}AT$ where

$T = \begin{pmatrix}\text{Cos}\theta & \text{Sin}\theta\\-\text{Sin}\theta & \text{Cos}\theta\end{pmatrix}$. Find the value of θ which gives this diagonal transformation.

5. (a) Show that rotation about the z-axis form a subgroup of SO (3). Is it an invariant subgroup?

(b) Prove that the number of distinct elements in a Coset of a subgroup is the same as the number of elements in the subgroup. 5+5

6. (a) $G = \{1, -1, i, -i\}$

Find different classes of this group G.

(b) Show that a Hermitian matrix elements is Hermitian under unitary similarity transformation.

(c) Define Lie group and its generator. 4+4+2