2015

M.Sc. Part–II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—IX (OR/OM)

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper : OR

(*Advanced Optimization and Operations Research – I*)

Answer Q. No. *11* and any *six* from the rest.

1. (a) Using decomposition principle reduce the following problem to an elegant form of LPP which can solve by simplex or modified simplex method :

(Turn Over)

$$\text{Max } f = 8x_1 + 3x_2 + 8x_3 + 6x_4$$

subject to

$$\begin{aligned} 4x_1 + 3x_2 + x_3 + 3x_4 &\le 16 \\ 4x_1 - x_2 + x_3 &\le 12 \\ x_1 + 2x_2 &\le 8 \\ 3x_1 + x_2 &\le 10 \\ 2x_3 + 3x_4 &\le 9 \\ 4x_3 + x_4 &\le 12 \\ x_j \ge 0,\ j = 1, 2, 3, 4. \end{aligned}$$ 8

(b) State and prove Fritz-John saddle point necessary optimality theorem. When does the theorem fail? 6+2

2. (a) Define constraint qualification and explain the cause of their introduction in the theory of non-linear programming problem. Discuss about two such constraint qualification. 8

(b) Use the artifical constraint method to find the initial basic solution of the following problem and then apply the dual simplex algorithm to solve it

Maximize $Z = 2x_1 - 3x_2 - 2x_3$

Subject to

$$\begin{aligned} x_1 - 2x_2 - 3x_3 &= 8 \\ 2x_1 + x_3 &\le 10 \\ x_2 - 2x_3 &\ge 4 \\ x_1, x_2, x_3 &\ge 0. \end{aligned}$$ 8

3. (a) State and prove Wolfe's duality theorem. 5

(b) Define the following :

(i) the (primal) quadratic minimization problem

(ii) the quadratic dual (maximization) problem 4

(c) Use Beale's method to solve the following quadratic programming problem

Maximize $Z = 2x_1 + 3x_2 - 2x_2^2$

subject to the constraints

$x_1 + 4x_2 \le 4,\ x_1 + x_2 \le 2$ and $x_1, x_2 \ge 0$ 8

4. (a) Let θ be a numerical differentiable function on an open convex set $\Gamma C R^n$. Prove that θ is convex on Γ if and only if $\theta(x^2) - \theta(x^1) \ge \Delta\theta(x^1)(x^2 - x^1)$ for each $x^1, x^2 \in \Gamma$. 5

(b) Let $X°$ be an open set in R^n, and let θ and g be defined on $X°$. Find the conditions under which

(i) A solution $(\bar{x}, \bar{r}_0, \bar{r})$ of the Fritz John saddle point problem FJSP is a solution of the Fritz-John stationary point problem. FJP and conversely.

(ii) A solution $(\bar{x}, \bar{u})$ of the Kuhn-Tucker saddle point problem KTSP is a solution of the Kuhn-Tucker stationary point problem KTP and conversely. 3

(c) Consider the following quadratic programming problem

Maximize $Z = 2x_1 + 3x_2 - x_1^2 - x_2^2$

subject to $x_1 + x_2 \le 2$

$x_1, x_2 \ge 0$

(i) Use the Kuhn-Tucker conditions to derive an optimal solution.

(ii) Solve the problem by Wolfe's modified simplex method.

5. (a) State and prove Tucker's lemma on non-linear programming. 8

(b) Derive the Kuhn-Tucker conditions for quadratic programming problem. Under what condition, these conditions will be necessary and sufficient. 8

6. (a) Write down the revised simplex algorithm to solve a LPP. How it is differ from original simplex method ? 6+2

(b) The optimal solution of the LPP

Maximize $Z = 3x_1 + 5x_2$

subject to $x_1 + x_2 \le 1$

$2x_1 + 3x_2 \le 1$

and $x_1, x_2 \ge 0$

is contained in the table

X_B	C_B	b	y_1	y_2	y_3	y_4
x_3	0	$\frac{2}{3}$	$\frac{1}{3}$	0	1	$-\frac{1}{3}$
x_2	5	$\frac{1}{3}$	$\frac{2}{3}$	1	0	$\frac{1}{3}$
		$\frac{5}{3}$	$\frac{1}{3}$	0	0	$\frac{5}{3}$

Find the range of changes of C_1 and C_2 for which the optimal solution remain optimal solution. 2+2+4

7. (a) Graphically solve the following goal programming problem :

A firm manufactures two products X and Y. The profits are Rs. 30 and Rs. 40 respectively for each kg of products. The firm has two machines and the processing time required for each machine on each product is given in the following table. Machines A and B have 180 and 160 hours respectively

		Products X	Products Y
Machines	A	60	30
	B	40	40

The management of the firm has established the following goal priorities :

Priority 1 : To maximize the profit

Priority 2 : To meet production goal of 2.5 kg of X and 1.5 kg of Y. 9

(b) Use Golden section method maximize

$$f(x) = \begin{cases} 3x/2, & x \le 2 \\ 5 - x, & x > 2 \end{cases}$$

in the interval [0, 6] upto six experiments. 8

8. (a) (i) Construct the Gomorian construint for solving mixed integer L.P.P. 5

(ii) Ignoring integer restrictions, the optimal simplex table of the following mixed integer L.P.P. :

Maximize $Z = 7x_1 + 9x_2$

subject to the constraints

$-x_1 + 3x_2 \le 6$

$7x_1 + x_2 \le 35$

$x_1, x_2 \ge 0$ and x_1 is integer.

		C_j	7	9	0	0
C_B	Y_B	X_B	y_1	y_2	y_3	y_4
9	y_2	$x_2 = \frac{7}{2}$	0	1	$\frac{7}{22}$	$\frac{1}{22}$
7	y1	$x_1 = \frac{9}{2}$	1	0	$-\frac{1}{22}$	$\frac{3}{22}$
		$Z_B = 63$	0	0	$\frac{28}{11}$	$\frac{15}{11}$ ← Δ_j

Construct the Gomorian constraint of the above problem. 3

(b) Use revised simplex method to solve the following L.P.P. :

Maximize $Z = 2x_1 + 4x_2 + 7x_3$

subject to $x_1 + x_2 + 2x_3 \le 12$

$8x_1 + 2x_2 + 3x_3 \le 10$

$x_1, x_2, x_3 \ge 0$ 8

9. (a) Let A be a real symmetric matrix of order n and the quadratic function

$$Q(X)=\frac{1}{2}X^TAX+B^TX+C$$

is minimized sequentially once along each of a set of n A-conjugat directions, show that the global minimum of Q(X) will be located at a before the nth set regardless of the starting point and the order in which the directions are used. 8

(b) Use branch and bound method solve the following LPP :

Maximize $Z = 7x_1 + 9x_2$

subject to the constraints

$$-x_1 + 3x_2 \le 6$$
$$7x_1 + x_2 \le 35$$

$x_1, x_2 \ge 0$ and integers. 8

10. (a) What is post optimality analysis? Discuss the following effect of changes in the optimal table. (i) addition of a new variable (ii) deletion of a variable. 2+3+3

(b) Using Davidon-Fletcher-Powell method, solve the following problem :

Minimize $f = x_1^2 + 7x_2^2 + 2x_1 - 14x_2$

with initialization $x^{(0)} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$. 8

11. Answer any *one* of the following : 1×4

(a) (i) What do you meant by the term "golden section". 2

(ii) How does goal programming differ from linear programming? 2

(b) Write a short note on "Theorems of the Alternative." 4

Special Paper : OM

Answer Q. No. *11* and any *six* questions from the rest.

1. (a) Find the condition of stability of equilibrium of a stratified fluid and hence explain the significance of the Brunt-Vaisala frequency. 8

(b) Obtain an expression of the Brunt-Vaisala frequency for the following cases :

(i) in a homogeneous layer where salinity and temperature vary little with depth.

(ii) in layers where temperature and salinity variation with depth are large.

2. Give a definition of salinity of sea-water. Derive the following relations :

(i) $C_v = C_p + T\left\{\left(\frac{\partial\tau}{\partial p}\right)^2 \Big/ \left(\frac{\partial\tau}{\partial p}\right)\right\}$;

(ii) $\Gamma = \frac{T}{C_p} \cdot \frac{\partial\tau}{\partial p}$;

(iii) $\Gamma_n = K_T - \Gamma.\alpha = K_T(C_v/C_p)$.

Where symbols have their usual meanings. 16

3. (a) Assuming that the mass exchange process across the free ocean surface $F(\bar{r}, t) = 0$ amount to a flux b of pure water in unit time per unit area, obtain the boundary conditions at the free ocean surface. 8

(b) Assuming that sea-water is a two component mixture of salt and pure water, show that the principle of conservation of mass leads to the pair of equations

$$\frac{D\rho}{Dt} + \rho \operatorname{div} \vec{q} = 0$$

$$\rho \frac{Ds}{Dt} = -\operatorname{div} \vec{I}_s$$

Where symbols have their usual meanings. 8

4. (a) Obtain the equation of motion of sea-water in the following form

$$\frac{d\vec{q}}{dt} = \vec{F} + 2\vec{q} \times \vec{r} - \frac{1}{\rho}\vec{\nabla} p + \frac{1}{\rho}(\lambda + \mu)\,\nabla(H) + \gamma\nabla^2 \vec{q}$$

where $(H) = \vec{\nabla}.\vec{q}$ and symbols have their usual meanings. 8

(b) Discuss free waves in an incompressible exponentially stratified rotating spherical fluid layer in the ocean. 8

5. Write down the equations for small amplitude wave motion in the ocean and deduce from these equations, the energy conservation equation. 16

6. Define absolute vorticity. Deduce the general vorticity equation of motion.

Show that equations of inertia currents are

$$\frac{Du}{Dt} - fv = 0\,;$$

$$\frac{Dv}{Dt} + fu = 0.$$

2+8+6

7. Show that plane Rossby waves pattern moves west ward with a speed $\frac{\sigma}{K}$. Hence deduce that the group velocity vector makes an angle 2α with x-axis. Find the magnitude of the energy flux. 6+5+5

8. In case of shallow water model, show that the volume of fluid column remains constant throughout the motion. 16

9. Develop Munk's theory of the wind driven ocean circulation. 16

10. Answer any *one* question : 4

(i) Discuss Rossby Number ;

(ii) Define adiabatic temperature.

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