2015

M.Sc. Part–II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—VIII

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group-A

Answer Q. No. *1* and any *three* from the rest.

1. Answer any *two* of the following : 2×4

(a) If f(x) has the Fowrier transform f(s) then prove that f(x) cos ax has the Fourier transform

$$\frac{1}{2}\left[F(s-a)+F(s+e)\right].$$ 4

(b) What do you mean by Fredholm Alternative in Integral equation ? Define eigen value and eigen function of an Integral equation. 4

(c) When a function f(t) is said to be of exponential order $O(e^{at})$ when $t > 0$? If f(t) is of exponential order ; what extra condition is needed for its Laplace transform to exist ? If the two conditions are met, find the domain in complex p-plane where Laplace transform of f(t) exists. 4

2. (a) Consider the following boundary value problem in the half plane $y > 0$, described by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad -\infty < x < \infty, \; y > 0$$

with boundary conditions $u(x, 0) = f(x), \; -\infty < x < \infty$,

u is bounded as $y \to \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. Solve the above problem. 7

(b) What do you mean by Integral equation ? Reduce general boundary value problem

$$\frac{d^2y}{dx^2} + A(x)\frac{dy}{dx} + B(x)\, y(x) = g(x)$$

to the integral equation with boundary conditions $y(a) = C_1$ and $y(b) = C_2$ and find kernel k(x, t) respectively. 7

3. (a) Define good function and fairly good function. Let $\gamma(x)$ be a good function and $f(x) = \int_{-\infty}^{x} \gamma(t)\, dt$. Then show that f(x) will be good function iff $\int_{-\infty}^{\infty} \gamma(t)\, dt = 0$. 6

(b) State and prove Parseval's identity on Fourier transform. Use Parseval's relation for Fourier cosine transform to prove the following

$$\int_0^{\infty} \frac{\sin \lambda t \sin \mu t}{t^2}\, dt = \frac{\pi}{2} \min \{\lambda, \mu\}.$$ 5+3

4. (a) State Bromuich's Integral formula. Using it, evaluate

$$L^{-1}\left\{\frac{p}{(p+1)^3\,(p-1)^2}\right\}$$

Where L^{-1} denotes for an inverse Laplace transform. 5

(b) Solve the integral equation

$$\int_0^x \frac{\varphi(t)}{(x-t)^{1/2}}\,dt = x + x^2.$$ 4

(c) Prove that all the eigen values of a regular Sturm-Lionville problem with positive weight function, are real. 5

5. (a) If the integral $\int_0^\infty f(r)\,dr$ is absolutely convergent and f(r) is continuous in the neighbourhood of r, then prove that $f(r) = \int_0^\infty \propto F_0(\propto)\,J_0(\propto r)\,d\propto$, where $F_0(\propto)$ is

the Hankel transform of order zero of the function f(r) and $J_0(\propto r)$ is the Bessel function of order zero. 6

(b) Solve the following boundary value problem using Green's function

$$\frac{d^2y}{dx^2} + y = x$$

with boundary conditions $y(0) = y(\pi/2) = 0$. 6

(c) Prove that the Fourier transform of a function ; if it exists, is bounded. 2

6. (a) If a function $\frac{f(t)}{t}$ satisfies the conditions of its Laplace transform and $L\{f(t)\} = F(p)$, which exists for real (p) > ν, then prove that

$$L\left\{\frac{f(t)}{t}\right\} = \int_p^\infty F(u)\,du$$

where ν is an exponential order. 5

(b) Find the first order Hankel transform of

$$f(r) = \frac{1}{r} e^{-ar}.$$ 4

(c) Evaluate $L\left\{\int_0^t \frac{\sin u}{u} du\right\}$ by the help of Initial value theorem. 5

Group-B

(Elements of Optimization and Operations Research)

[For the students whose special paper is OM]

[Marks : 50]

Answer Q. No. 7 and any *three* from the rest.

7. Define convex and concave function. 2

or

What is mixed integer programming ? Write down at least one method name that solve mixed integer programming problem. 2

8. (a) Using Kuhn-Tucker necessary conditions solve the following problem

Minimize $f(x) = x_1^2 + x_2^2 + x_3^2$

Subject to
$$2x_1 + x_2 \le 5$$
$$x_1 + x_3 \le 2$$
$$x_1 \ge 1$$
$$x_2 \ge 2$$
$$x_3 \ge 0$$ 8

(b) Write down the revised simplex procedure to solve a LPP. What are the advantages of revised simplex method over the original simplex method. 6+2

9. (a) Using Dynamic programming technique, show that

$$Z = \sum_{i=1}^{n} p_i \log p_i$$

subject to the constraints

$$\sum_{i=1}^{n} p_i = 1 \text{ and } p_i > 0 \text{ for all } i$$

is minimum when $p_1 = p_2 = \dots = p_n = \frac{1}{n}$. 8

(b) Find the economic lot size for the inventory model with finite replenishment rate, uniform finite demand and zero lead time so that the total average cost is minimum. 8

10. (a) Solve the following quadratic programming problem by using Wolfe's method :

Maximize $Z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

subject to the constraints

$x_1 + 2x_2 \le 10,\ x_1 + x_2 \le 9,\ x_1, x_2 \ge 0.$ 8

(b) Find the optimum order quentity for a product for which the price breaks are as follows :

Range of quantity to be purchased (Q)	Purchase cost per unit (Rs.)
$0 < Q < 100$	20.00
$100 \le Q < 200$	18.00
$200 \le Q$	16.00

The monthly demand for the product is 400 units. The storage cost is 20% of the unit cost of the product and the cost of ordering is Rs. 25.00 per order. 8

11. (a) Solve the following LPP using revised simplex method

Maximize $Z = 4x_1 + x_2$

subject to $2x_1 + 7x_2 \le 21$

$7x_1 + 2x_2 \le 21$

$x_1, x_2 \ge 0$ 8

(b) The optimal solution of the following LPP

Maximize $Z = 2x_1 + x_2 + 4x_3 - x_4$

subject to $x_1 + 2x_2 + x_3 - 3x_4 \le 8$

$-x_2 + x_3 + 2x_4 \le 0$

$2x_1 + 7x_2 - 5x_3 - 10x_4 \le 21$

and $x_1, x_2, x_3, x_4 \ge 0$

is contained in the following simplex table

X_B	C_X	b	y_1	y_2	y_3	y_4	y_5	y_6	y_7
x_1	2	8	1	0	3	1	1	2	0
x_2	1	0	0	1	–1	–2	0	–1	0
x_7	0	5	0	0	–4	2	–2	3	1

When b is changed to $(3, -2, 4)^T$ make the necessary corrections in the optimum table and show that the resulting problem has no feasible solution. 8

12. (a) How does quadratic programming problem differ from the non-linear programming problem ? Discuss the

Beale's method to solve the quadratic programming problem. 2+6

(b) Solve the following L.P.P. by Branch and Bound method :

Maximize $Z = 6x_1 + 8x_2$

subject to the constraints

$2x_1 + 3x_2 \le 16$

$7x_1 + 2x_2 \le 14$

$x_1, x_2 \ge 0$, x_1, x_2 are integers. 8

Group-B

(Dynamical Oceanology and Meteorology)

[For the students whose special paper is OR]

[Marks : 50]

Answer Q. No. *12* and any *three* from the rest.

7. (a) Find the equation of motion of an air parcel in the cartesian co-ordinates in the atmosphere. 8

(b) Derive the variation of pressure with attitude in the hydrostatic atmosphere when temperature decreases at a constant rate with increasing attitude. 4

(c) Derive the adiabatic lapse rate of dry air. 4

8. (a) Obtain the atmospheric energy equation and interpret each term. 8

(b) Derive the hypsometric equation in the atmosphere. 4

(c) Derive an expression for the density ρ of an air parcel at pressure p if it is adiabatically expands from a level where pressure and density are p_r and ρ_s respectively. 4

9. (a) Derive the hydrostatic equation in the atmosphere. 4

(b) Define relative humidity. Find a relation between mixing ratio and specific humidity. 4

(c) Find the condition of stability of equilibrium of a stratified fluid and hence explain the significance of the Brunt-Vaisala Frequency. 8

10. Define the general momentum equation of motion of the currents. Hence, deduce the hydro-static equation as

$$\frac{\partial p}{\partial z} = -\rho g \, . \qquad 8+8$$

11. Give, a definition of salinity of sea-water. Derive th following :

(i) $C_v = C_p + T\left\{\left(\frac{\partial\tau}{\partial p}\right)^2 \Big/ \left(\frac{\partial\tau}{\partial p}\right)\right\}$,

(ii) $\Gamma = \frac{T}{C_p} \cdot \frac{\partial\tau}{\partial p}$

(iii) $\Gamma_n = K_T - \Gamma\propto = K_T\left(C_v/C_p\right)$

Where symbols have their usual meanings.

12. Define frontal surface in the atmosphere. 2
