2015

M.Sc. Part–I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—V

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

(*Mechanics of Continuous Media*)

[*Marks : 50*]

Answer *Q. No. 6* and any *three* questions from the rest.

1. (a) Give the geometrical interpretation of small strain tensors. 8

(b) State and prove Kelvin's circulation theorem and hence show that an irrotational motion is always irrotational for homogeneous incompressible fluid in a conservative force field. 8

2. (a) Prove that the extreme values of normal stress at a point of a continuum are principal stresses. 8

(b) If $F(x, y, z, \in) = 0$ remains a moving boundary surface of a fluid motion, find the kinematical boundary condition. 8

3. (a) Obtain the equation of conservation of mass both in Eulerian co-ordinate system and Lagrangian co-ordinate system. 8

(b) If the equation characterizing the deformation are given by :

$$x_1 = X_1 + \in X_2$$
$$x_2 = X_1 + \in X_3$$
$$x_3 = X_3 + \in X_2$$

where (X_1, X_2, X_3) are the co-ordinates of a point with respect to a fixed axes in space, determine the infinite stain tensor. 8

4. (a) Explain the inversion of Hook's law, Young's modulus, Poisson's ratios and modulus of rigidity. 8

(b) Define a doublet in a fluid system. Then find the velocity potential due to a doublet in a homogeneous incompressible fluid. 8

5. (a) Derive the Cauchy's integrals of Lagrange equation of motion of perfect fluid in terms of vorticity. 8

(b) The state of stress at a point is given by :

$$\sigma_{ij} = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 4 & 0 \\ 3 & 0 & -1 \end{pmatrix}$$

(i) Find the stress vector at the point on the plane whose normal is in the direction $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$.

(ii) Determine the magnitude of the normal and shearing stress on this plane. 6

(c) What is the concept of stream function ? 2

6. (a) Define principal stress and principal directions of stress. 2

or

(b) Define rigid and strain deformation. 2
