2015

M.Sc. Part–I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—IV

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answer to questions of each group in Separate answer booklet.

Group—A

(*Principles of Mechanics*)

[*Marks* : 50]

Answer *Q. No. 1* and any *three* questions from the rest.

1. Answer any *one* question : 2

(a) State the basic postulates of special theory of relativity.

(b) What do you mean by inertial frame ? Give an example.

2. (a) Show that the motion of a system of particles is equivalent to the motion of its centre of mass.

(b) What is the effect of the Coriolis force on a particle falling freely under the action of gravity ?

(c) Calculate the order and magnitude of Centripetal acceleration of a particle on earth's surface .(*Assume the radius of earth is 6400 km*).

A particle is moving with velocity 20 cm/sec. What is the maximum Coriolis acceleration ? 4+8+4

3. (a) Deduce the Lagrange's equation of motion for a system of particles in case of connected holonomic system. 8

(b) A particle of mass m moves in one dimension such that it has the Lagrangian

$$L = \frac{m^2\dot{x}^4}{12} + m\dot{x}^2V(x) - V^2(x),$$

where V is some differentiable function of x. Find the equation of motion for x(t) and describe the physical nature of the system on the basis of this equation. 8

4. (a) Find the Euler's dynamical equations of motion of a rigid body when rotating with an angular velocity w about a fixed point. 8

(b) A body moves under no forces about a point O, the principal moments of inertia at O, being 6A, 3A and A. Initially, angular velocity of the body has the components $w_1 = n$, $w_2 = 0$, $w_3 = 3n$ about the principal axis.

Show that at any limit t, $w_2 = -\sqrt{5}n \tanh \sqrt{5}nt$ and ultimately body rotates about the mean axis. 8

5. (a) State and explain the Hamilton's principle and derive Lagrange's equation of motion from it. 8

(b) Let X and Y be two dynamical variables and their Poisson bracket be denoted by [X, Y]. Prove that

(i) $[X+Y, Z] = [X, Z] + [Y, Z]$,

(ii) $[X, Y]_{q, p} = [X, Y]_{Q,P}$,

the symbols have their usual meanings. 3+5

6. (a) Derived the Lorentz transformation equations in connection with special theory of relativity. 10

(b) Derive the necessary and sufficient condition for a transformation to be canonical.

Test whether the transformation :

$$q = \sqrt{\frac{p}{c}} \sin Q, \; p = \sqrt{mpc} \cos Q$$

is Canonical. 3+3

Group—B

(Partial Differential Equation)

[Marks : 50]

Answer *Q. No. 1* and any *three* from the rest.

1. (a) Define domain of dependence for the Cauchy problem of homogeneous wave equation. 2

Or

(b) Obtain a partial differential equations from the following relation F(x, y, z, a, b) = 0, where 'a' and 'b' are arbitrary constants. 2

2. (a) Find the complete integral of the equation :

$px^5 - 4q^3x^2 + 6x^2z - 2 = 0.$ 8

(b) Find the integral surface of the partial differential equation :

$(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$

passing through the curve $xz = a^3$, $y = 0$. 8

3. (a) Consider the equation :

$$u_{xx} + 4u_{xy} + u_x = 0$$

(i) Find the canonical form of the equation.

(ii) Find the general solution u(x, y) of the equation. 8

(b) Show that the Green's function for the equation :

$$\frac{\partial^2 u}{\partial x \partial y} + u = 0$$

is $v(x,y\ ;\ \xi,\eta) = J_0\sqrt{2(x-\xi)(y-\eta)}$

where J_0 denotes Bessel's function of first kind and order zero.

4. (a) Establish Green's first identity. Using this identity, show that if a harmonic function vanishes at all points on the boundary of a bounded and smooth domain D, then it is identically zero in D. 6

(b) Solve the Laplace equation in the square $0 < x, y < \pi$ subject to the Dirichlet condition $u(x, 0) = 107$, $u(x, \pi) = u(0, y) = u(\pi, y) = 0$. 6

(c) Let u be a harmonic function on the whole plane such that $u = 3\sin(2\theta) + 1$ on the circle $x^2 + y^2 = 2$. Without finding the concrete form of the solution, find the value of u at the origin. 4

5. (a) Find the solution of the one-dimensional diffusion equation satisfying the initial condition :

$$T(x, 0) = x(a - x),\ 0 < x < a,$$

the regularity condition that T is bounded as $t \to \infty$ and the boundary condition

$$\frac{\partial}{\partial x}T(0,t) = \frac{\partial}{\partial x}T(a,t)$$

for all $t \geq 0$. 8

(b) Find the solution of the following Partial differential equation using separation of variables method :

$$u_{xx} - u_y + u = 0.$$ 8

6. (a) Derive the D'Alembert's formula for the Cauchy problem of the non-homogeneous wave equation. 8

(b) Show that the Green's function for Dirichlet problem is symmetric. 4

(c) Show that if u solves the Neumann problem for Poisson's equation, then any other solution is of the form $v = u + c$ for some real number c. 4