2015

M.Sc. Part–I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—III

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Write the answer to questions of each group in Separate answer booklet.

Group—A

(*Probability and Statistics*)

[*Marks : 30*]

Answer any *two* questions : 2×15

1. (a) Derive the differential equations for pure birth process and then solve these by certain conditions to be stated by you. Also find the mean population size under this process. 10

(b) State and prove Chapman-Kolmogorav equation. 5

2. (a) What do you mean by Galton-Watson Branching process ? Prove that :

$P_n(S) = P_{n-1}(P(S))$ *and*

$P_n(S) = P(P_{n-1}(S))$.

The symbols have their usual meanings. Also state the properties of generating function of Branching process.

2+5+1

(b) Define persistent and transient states. Prove that the state j is persistent iff

$$\sum_{n=0}^{\infty} p_{jj}^{(n)} = \infty,$$

where $p_{jj}^{(n)}$ is the probability that it reaches from state i to j, not necessarily for the first time, after n transitions. 7

3. (a) What is the difference between Markov process and Markov chain ? 3

(b) What do you mean by Stochastic process ? Define state and state space with examples. 4

(c) Write the important properties of multiple correlation coefficient. 3

(d) What do you mean by partial correlation and multiple correlation coefficient of three variables ? Establish the relationship between them. 5

Group—B

(*Numerical Analysis*)

[*Marks : 40*]

Answer *Q. No. 4* and any *three* from the rest.

4. Prove the following relations : 4

(a) $1+\delta^2\mu^2 = \left(1+\frac{\delta^2}{2}\right)^2$

(b) $hD \equiv \sinh^{-1}(\mu\delta)$

5. (a) Deduce Stirling's interpolation formula for odd number of equispaced arguments. 8

(b) Use inverse Lagrange's interpolation to find a root of the equation $x^3 - 3x + 1 = 0$. 4

6. (a) Let $y = f(x)$ be a function defined on [a, b]. Describe a method to approximate this function in a polynomial degree n with the help of orthogonal polynomials. 6

(b) Starting from $f(x) = 1, x, x^2, x^3, x^4, x^5$, deduce the following three point formula :

$$\int_{-1}^{1} f(x)dx = \frac{1}{9}\left[5f\left(-\sqrt{0.6}\right) + 8f(0) + 5f\left(\sqrt{0.6}\right)\right],$$

provided the formula is exact for these functions. 6

7. (a) Describe Newton-Raphson method to solve the following non-linear equations $f(x, y) = 0$ and $g(x, y) = 0$.

Using this method to solve the system

$x^2 - 2x - y + 0.5 = 0$, $x^2 + 4y^2 - 4 = 0$ with the starting value $x_0 = 2{\cdot}0$, $y = 0{\cdot}25$. 4+4

(b) Describe Runge-Kutta method to solve the following pair of differential equations :

$$\frac{dy}{dx} = f(x, y, z), \quad \frac{dz}{dx} = g(x, y, z)$$

with initial conditions $x = x_0$, $y(x_0) = y_0$, $z(x_0) = z_0$. 4

8. (a) Describe Jacobi's method to find all eigenvalues and eigenvectors of a real symmetric matrix. 8

(b) Suppose $y = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \frac{x^4}{8!} - \cdots$

Economize this series if the fourth-decimal places is not to be affected, near $x = 1$. 4

9. (a) Describe Crank-Nicolson implicit method to solve an one-dimensional heat equation. 6

(b) Use Milne's predictor-corrector method to find the value of $y(0{\cdot}10)$ for the initial value problem :

$$\frac{dy}{dx} = y \sin x, \; y(0)=1$$

by taking $h = 0{\cdot}10$. 6

Group—C

(*Introduction to Computing*)

[*Marks : 30*]

10. Answer any *six* questions : 6×5

(i) Design a logic circuit of an overheat alarm for an oil fired steam boiler. In this system there are three sensors. One of them monitors the water temperature in the boiler, another monitors the chimney temperature and the other follows on-off state of the burner. An alarm signal should be generated whenever the burner is on and either the chimney or water temperature is too hot.

(ii) What are the advantages and disadvantages of C-pointer? What type of operators can be implemented on pointers? Explain through examples.

(iii) Write short notes on the following :

(a) scanf function ;

(b) while statement.

(iv) Write a program in C to find the sum of all digits in a number using a function.

(v) Explain BCD Coding. Write down the algorithm of BCD addition. Add the following BCD numbers :

10110110 + 11001011

(vi) (a) What is conditional operator? Give an example.

(b) How is an array passed to a function? Give an example.

(c) How is an array of structure initialized?

(vii) What do you understand by 'Call by Value' and 'Call by reference'? Explain these two in swapping two values through two programs.

(viii) What do you mean by 'recursion'? Write a program in C to evaluate the factorial of an unsigned integer number using recursion.

(ix) Write a program to count the no. of vowels in a word using switch statement.

(x) Write a program to check whether a number is prime or not, using a function.

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