2015

M.Sc. Part–II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—X (OR/OM)

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper : OR

(*Advanced Optimization and Operations Research – II*)

Answer Q. No. *11* and any *six* from the rest.

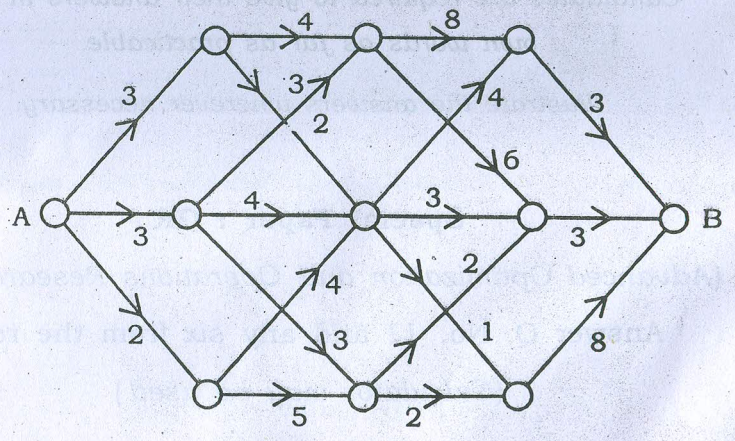
(*Calculator may be used*)

1. (a) Use dynamic programming technique, show that

$$\sum_{i=1}^{n} p_i \log p_i \text{, subject to } \sum_{i=1}^{n} p_i = 1$$

is minimum when $p_1 = p_2 = \ldots = p_n = \frac{1}{n}$.

(b) Use dynamic programming method to find the shortest path from the vertex A to the vertex B along edges joining various vertices lying between A and B shown in the flowing figure.



The numbers adjacent to edges represent the weights of the edges. 8+8

2. (a) Find the optimum order level for a manufacturing model with shortage, finite rate of replenishment, constant demand and infinite production planning horizon.

(b) The demand for an item in a company is 18000 units per year. The company can produce the item at a rate of 3000 per month. The cost of one set-up is Rs. 500 and the holding cost of one unit per month is Rs. 0·15. The shortage cost of one unit is Rs. 20 per month. Determine the optimum manufacturing quantity and the shortage quantity. Also determine the manufacturing time and the time between setups. 10+6

3. (a) Write down the rules to draw a network for project management system.

(b) What is critical path ? What are the main features of it ?

(c) A small project consist of seven activities, the details of which are given below :

Activity	Time estimates			Predecessor
	t_o	t_m	t_p	
A	1	1	7	—
B	1	4	7	—
C	2	2	8	—
D	1	1	1	A
E	2	5	14	B
F	2	5	8	C
G	3	6	15	D, E
H	1	2	3	F, G

Find the critical path. What is the probability that the project will be completed by 18 weeks ?

4+4+8

4. (a) Define transient state and steady state in queuing system.

Write a brief note about the service discipline in queuing system.

(b) Obtain the probability p_n of n customer in the system for the (M/M/C) : (N/FCFS/∞) queuing model.

6+10

5. (a) A firm has a single channel service station with the following arrival and service time probability distributions :

Inter-arrival time (minutes)	*Probability*	*Service time (minutes)*	*Probability*
10	0·10	5	0·08
15	0·25	10	0·14
20	0·30	15	0·18
25	0·25	20	0·24
30	0·10	25	0·22
		30	0·14

The customer's arrival at the service station is a random phenomenon and the time between the arrival varies from 10 minutes to 30 minutes. The service time varies from 5 minutes to 30 minutes. The queuing process begins at 10 am and proceeds for 8 customers. An arrival goes to the service facility immediately if it is free. Otherwise it will wait in a queue. The queue discipline in first come-first-served. If the attendant's wages are Rs. 10 per hour and the customer's waiting time costs Rs. 15 per hour, then would it be an economical proposition to engage a second atendant ? Answer using Monte-Carlo simulation technique. The random members for inter-

arrival and service times are 10, 20, 30, 80, 70 and 75, 30, 65, 35, 80, 90.

(b) Use the formula $X_{n+1} = (ax_n) \text{ (modm)}$ to calculate 8 random numbers starting from $x_0 = 2738$, $a = 65539$, and $m = 100$. 10+6

6. (a) What do you mean by memoey less channel and channel matrix? 4

(b) Prove that the entropy function $H(p_1, p_2, \ldots, p_n)$ is continuous and symmetric. 4

(c) Prove that $H(X, Y) = H(X/Y) + H(Y)$
$= H(Y/X) + H(X)$ where $H(X) \geq H(X/Y)$. 8

7. (a) In a system, there are n number of components connected in parallel with reliability

$R_i(t) = i = 1, 2, \ldots, n$. Find the reliability of the system. If $R_1(t) = R_2(t) = \ldots R_n(t) = e^{-\lambda t}$, then what will be the expression of system reliability?

The failure rate of an electronic subsystem is 0·0005 hrs. If an operational period of 500 hrs, with probability of success P(s) = 0·95 is derived. What label of parallel redundancy is needed? 5+3

(b) What is optimal control? Define control and state variables in connection with optimal control. Find the stationary path $x = x(t)$ for the functional

$$J = \int_0^1 \left[1 + \left(\frac{d^2x}{dt^2}\right)^2\right] dt$$

subject to the boundary conditions $x(0) = 0$, $\dot{x}(0) = 1$, $x(1) = 1$, $\dot{x}(1) = 1$. 4+4

8. (a) Using Geometric programming technique minimize the following

$$z = 5x_1x_2^{-1} + 2x_1^{-1}x_2 + 5x_1 + x_2^{-1}$$

and find the optimal values of z ; x_1 and x_2. 8

(b) Find the sequence that minimizes the total time required in performing the following jobs on three machines in the order ABC. Processing times (in hours) are given in the following table :

Job :	*1*	*2*	*3*	*4*	*5*
Machine A :	8	10	6	7	11
Machine B :	5	6	2	3	4
Machine C :	4	9	8	6	5

8

9. (a) A computer contains 10,000 resistors. When any resistor fails, it is replaced the cost of replacing a resistor individually is Re. 1 only. If all the resistors are replaced at the same time, the cost per resistor would be reduced to 35 paise. The percentage of surviving resistors say s(t) at the end of month t and the probability of failure P(t) during the month t are as follows :

t :	0	1	2	3	4	5	6
S(t) :	100	97	90	70	30	15	0
P(t) :	—	0·03	0·07	0·20	0·40	0·15	0·15

What is the optimal replacement plan ? 8

(b) Define separable game ? Discuss the canonical representation of the payoff function in a separable game. Show that the following function is cononical.

$$M(x, y) = xy - xe^y + 2x\cos y + 2e^x y + 3e^x e^y + e^x \cos y + 5\cos x.e^y - 3\cos x \cos y$$ 2+4+2

10. (a) Discuss the various types of failure in connection with reliability.

The data on the running costs per year and resale price of equipment A, whose purchase price is Rs. 2,00,000 are as follows :

Year :	1	2	3	4	5	6	7
Running Cost (Rs) :	30,000	38,000	46,000	58,000	72,000	90,000	1,10,000
Resale value (Rs) :	1,00,000	50,000	25,000	12,000	8,000	8,000	8,000

(i) What is the optimum period of replacement ?

(ii) When equipment A is two years old, equipment B, which is new model for the same usage, is available. The optimum period for replacement is 4 years with an average cost of Rs. 72,000. Should equipment A be changed with equipment B ? If so, when ? 2+8

(b) Derive Johnson's algorithm for processing n jobs through m machines in a sequence. 6

11. Answer any *one* question : 4

(a) What is continuous game ? State the fundamental theorem for continuous game.

(b) Write down the advantages and disadvantages of simulation.

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Special Paper : OM

(Dynamical Meteorology)

Answer any *five* questions. 5×15

1. (a) Deduce the Clausius-Clapeyron equation in the following form :

$$\frac{de_s}{dT} = \frac{e_s L}{R_v T^2}.$$

Interpret the equation. 7

(b) Define the wet bulb temperature. Derive Psychrometric equation to measure the actual vapor pressure in terms of dry bulb and wet bulb temperature. 4

(c) Derive the hydrostatic equation in the atmosphere. 4

2. (a) Derive the momentum equation of an air parcel in the atmosphere in spherical co-ordinates. 8

(b) Explain the convective available potential energy of an air parcel. 5

(c) Show that the potential temperature of an air parcel is invariant. 2

3. (a) Explain the slice method of stability analysis for an air panel in the atmosphere. 7

(b) Define mixing ratio and specific humidity. Then derive a relation between them. 4

(c) Show that the pressure tendency at the earth surface be zero when the air motion in geostrophic at all levels in the atmosphere. 4

4. (a) Derive the vorticity equation of an air parcel in the atmosphere and interpret each term. 7

(b) Find a relation between dew point and mixing ratio with total pressure of a moist air. 5

(c) Explain the convergence and divergence in the atmosphere. 3

5. (a) Derive the saturated adiabatic lapse rate of moist air and hence show that it is less than the dry adiabatic lapse rate. 7

(b) Explain the pressure trough at frouls' in the atmosphere. 4

(c) Derive the Poisson's equation in the atmosphere. 4

6. (a) Derive the formula to predict the potential temperature due to advection using finite difference method. 6

(b) Derive the relation from which saturation temperature can be obtained when saturation of air will be done by adiabatic ascent and for this case, estimate the height at which saturation will attend. 9

7. (a) Derive the circulation theorem for an air parcel in the atmosphere and interpret each term in the equation. 9

(b) How the cloud base can be derived for convective clouds ? 6

8. (a) What is geopotential surface ? Derive the geopotential thickness between two pressure levels in the atmosphere. 8

(b) What do you mean by gradient wind in the atmosphere ? Explain this wind to discuss cyclonic and anticyclonic motion both for normal and anomalous case in the northern hemisphere. 7

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