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Quantum Dynamics and Entanglement Properties of a Three-mode Atom-molecule Bose-Einstein Condensates

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ABSTRACT

We solve analytically the Hamiltonian of a three mode atom-molecule Bose-Einstein condensates (BECs) where the molecular mode is prepared through photoassociative Bose-stimulated Raman adiabatic passage (STIRAP). Using these solutions we investigate the time dependence population densities in all three modes and entanglement properties among the modes. We find that the stable molecular mode will remain highly populated over time as expected for STIRAP. We also observe the signature of entanglement between atomicBEC and excited molecular Bose-Einstein condensate (MBEC) mode as observed in two mode atom-molecule BECs, but the atomic BEC and stable molecular BEC are always separable.

Keywords:Entanglement, BEC, STIRAP, Sen-Mandal approach, MBEC, ETCR.

1. Introduction

The quantum states of BECs of weakly interacting dilute atomic gas has great theoretical and practical importance[1,2,3]. It is theoretically possible to prepare MBEC through Feshbach resonance [4] and photoassociation [5,6]. Two types of atom-molecule BECs are already reported: (i) two mode atom-molecule BECs and (ii) three-mode atom-molecule BECs. The dynamics and nonclassicalities of two-mode atom-molecule BECs considering the inter-mode interactionare reported [7,8]. Though the three-mode atom-molecule BECs are theoretically studied considering the intra-mode interactions [9],no theoretical work not yet attemptto study the entanglement properties of the system considering the inter-mode interactions. Here we investigate the population dynamics and entanglement properties of three-mode atom-molecule BECsprepared by STIRAP having inter-mode interactions.

STIRAP requires two laser pulses, one conduce free-bound photoassociation from atomic BEC to primarily excited molecular BEC and second laser pulse couples the excited molecular BEC mode to stable molecular BEC mode [6,9]. If the pulse is continuing for sufficiently long time then the stable atomic BEC is converted to a stable molecular BEC via an excited molecular BEC of insignificant population density[6,9]. Due to the minimal population density in the excited molecular state, the irreversible losses through photodissociation or spontaneous decay is negligible. In the present work we consider a three-mode atom-molecule BECs prepared by STIRAP having intermodal

interactions and maximal population in the stable molecular state to study the population dynamics, entanglement properties.

The present paper is organised as follows. In Sec. 2 we construct the Hamiltonian of the system of our present interest and solve the Heisenberg's equations of motion corresponding to this Hamiltonian. Sec. 3 describes the quantum dynamics of the system. In Sec. 4 we investigate the entanglement between any two modes and finally concluding in Sec. 5.

2. Quantum model and solution

For exact two photons resonance, the Hamiltonian of the three-mode atom-molecule BECs prepared by STIRAP [6] can be written as,

$$H = \delta b^{\dagger} b - \frac{\omega}{2}\left(a^{\dagger 2} b + a^{2} b^{\dagger}\right) - \frac{\epsilon}{2}\left(b^{\dagger} c + b c^{\dagger}\right). \quad (1)$$

where a, b and c are bosonicannihilation operator for atomic BEC mode$|1\rangle$, excited MBEC mode $|2\rangle$ and stable MBEC mode $|3\rangle$respectively and they commute with each other. The energy difference between the stable and excited levels is δ which can be tuned by external laser pulse. The parameters ω, ϵ correspond to atom-excited molecule and excited molecule-stable molecule interactions respectively.

The commutation relations between $a, a^{\dagger}; b, b^{\dagger}; c, c^{\dagger}$ are,

$$\begin{aligned} &\left[a, a^{\dagger}\right] = 1, \\ &\left[b, b^{\dagger}\right] = 1, \quad (2) \\ &\left[c, c^{\dagger}\right] = 1. \end{aligned}$$

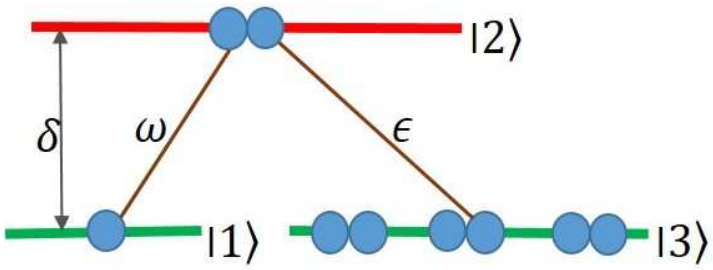

Figure 1. Schematic diagram of a three-mode atom-molecule BECs.

The maximal populations will be at stable molecular mode and for exact two photon resonance atomic and stable molecular mode will remain at same energy level as shown in Figure 1 [6]. Throughout the paper we take ħ=1. The time evolution of the system is described by the Heisenberg equations of motion which are;

$$\begin{aligned} &\dot{a}(t) = i\omega a^{\dagger}(t) b(t), \\ &\dot{b}(t) = -i\delta b(t) + i\frac{\omega}{2} a^{2}(t) + i\frac{\epsilon}{2} c(t), \quad (3) \\ &\dot{c}(t) = i\frac{\epsilon}{2} b(t). \end{aligned}$$

Among the above equations the first two are coupled nonlinear equations of field operators which cannot be exactly solvable in closed analytic form. So, we have go through approximate analytic solutions. It is established that among the available approximate analytic solutions method, the Sen-Mandal technique [10] give more accurate solutions [11] and well-agree with the exact numerical solution [8,11]. So, we solve the above equations using Sen-Mandal approach and the solutions are;

$$\begin{aligned} a(t) &= f_1 a(0) + f_2 a^{\dagger}(0)b(0) + f_3 a^{\dagger}(0)a^2(0) + f_4 a(0)b^{\dagger}(0)\mathrm{b}(0) \\ &\quad + f_5 a^{\dagger}(0)c(0), \\ b(t) &= g_1 b(0) + g_2 a^2(0) + g_3 \mathrm{c}(0) + g_4 b(0) + g_5 a(0)b^{\dagger}(0)\mathrm{b}(0), \quad (4) \\ c(t) &= h_1 c(0) + h_2 b(0) + h_3 a^2(0) + h_4 c(0). \end{aligned}$$

The time dependent parameters $f_i(i = 1,2,3,4,5)$,$g_i(i = 1,2,3,4,5)$and $h_i(i = 1,2,3,4)$are:

$$\begin{aligned} f_1 &= h_1 = 1, \\ f_2 &= 2g_2 = \frac{\omega}{\delta}G(t), \\ f_3 &= -\frac{f_4}{2} = -\frac{\omega^2}{2\delta^2}[G(t) - i\delta t], \\ f_5 &= 2h_3 = -\frac{\omega\epsilon}{2\delta^2}[G(t) - i\delta t], \\ g_1 &= e^{-i\delta t}, \quad (5) \\ g_3 &= h_2 = \frac{\epsilon}{2\delta}G(t), \\ g_4 &= -\frac{2\omega^2 + \epsilon^2}{4\delta^2}g_1[G(t) + i\delta t], \\ g_5 &= -\frac{\omega^2}{\delta^2}g_1[G(t) + i\delta t], \\ h_4 &= -\frac{\epsilon^2}{4\delta^2}[G(t) - i\delta t]. \end{aligned}$$

where$G(t) = \left(1 - e^{-i\delta t}\right)$.The above solutions are derived considering up to second order of the interaction parameters ω and εandvalid for any time value$t$ with the restriction $\omega t < 1$and $\varepsilon t < 1$. The present solutions satisfies the equal time commutation relations (ETCR):$[a(t), a^{\dagger}(t)] = 1, [b(t), b^{\dagger}(t)] = 1, [c(t), c^{\dagger}(t)] = 1$ and conservation law of total particle number$a(t)a^{\dagger}(t) + 2b(t)b^{\dagger}(t) + 2c(t)c^{\dagger}(t)$=constant.

3. Quantum dynamics

Initially we consider all the three states are coherent and almost all the particles are at stable molecular mode with negligible population at excited molecular state as expected from STIRAP. The composite initial state of the system can be written as:

$$|\psi(0)\rangle = |\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle. \quad (6)$$

where $|\alpha\rangle$,$|\beta\rangle$and $|\gamma\rangle$ are eigen states of a,$b$and $c$respectively.The operation of the annihilationoperators a, b and c operating on the composite system at $t = 0$ give corresponding complexeigen values α, β and γ respectively. So,

$$\begin{aligned} a(0)|\psi(0)\rangle &= \alpha|\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle, \\ b(0)|\psi(0)\rangle &= \beta|\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle, \quad (7) \\ c(0)|\psi(0)\rangle &= \gamma|\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle. \end{aligned}$$

Instantaneous particles number in the three states are,

$$\begin{aligned} N_a(t) &= |\alpha|^2 + |f_2|^2\left(|\beta|^2 + 2|\alpha|^2|\beta|^2 - \frac{1}{2}|\alpha|^4\right) + [f_1^* f_2 \alpha^{*2}\beta + f_1^* f_5 \alpha^{*2}\gamma + \text{c.c.}], \\ N_b(t) &= |\beta|^2 + |g_2|^2(|\alpha|^4 - 2|\beta|^2 - 4|\alpha|^2|\beta|^2) + |g_3|^2(|\gamma|^2 - |\beta|^2) \\ &\quad + [g_1 g_2^* \alpha^{*2}\beta + g_1 g_3^* \beta\gamma^* + g_2 g_3^* \alpha^2\gamma^* + \text{c.c.}], \quad (8) \\ N_c(t) &= |\gamma|^2 + |h_2|^2(|\beta|^2 - |\gamma|^2) + [h_1^* h_2 \beta\gamma^* + h_1^* h_3 \alpha^2\gamma^* + \text{c.c.}]. \end{aligned}$$

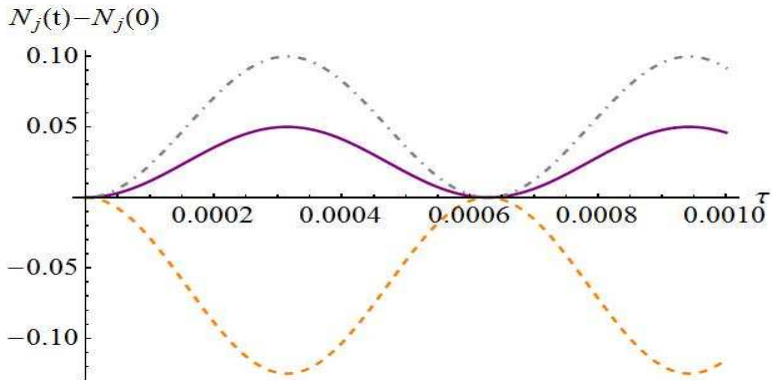

Figure 2. Variation of occupation number from their initial value with rescaled time$\tau = \omega t$, for $\alpha = \beta = 5, \gamma = 100, \delta = 10\ MHz, \omega = \epsilon = 1\ KHz$. The smooth line, dashed line, dot-dashed line for atomic, excited molecular and stable molecular mode respectively.

We plot$\left(N_j(t) - N_j(0)\right)$ for $j = a, b, c$with rescaled time $\tau = \omega t$ in Fig. 2 which show that,at any instant the particle number at the excited molecular mode increases (decreases) in expense of decrease (increase) in particle number at atomic mode and stable molecular mode and the occupation number does not change significantly due to

the interaction among the modes. The advantage of STIRAP processthat the final stable mode remain maximally populatedover time which is not disturbed by the introduction of inter-mode interactions.

4. Entanglement

We study the entanglement between any two modes by using Hillery-Zubairy criteria[12,13]. According to Hillery-Zubairy criterion,usually any two modes represented by annihilation operators$i$ and j is entangled at any instant t if,

$$\langle N_i(t)N_j(t)\rangle - \left|\langle i(t)j^{\dagger}(t)\rangle\right|^2 < 0. \quad (9)$$

and/or if,

$$\langle N_i(t)\rangle\langle N_j(t)\rangle - |\langle i(t)j(t)\rangle|^2 < 0. \quad (10)$$

Equation (8) & (9) are known as Hillery-Zubairy 1 (HZ-1) &Hillery-Zubairy 2 (HZ-2) criteria respectively.

To detect entanglement for our systemwe evaluate,

$$\begin{aligned}
&\langle N_a(t)N_b(t)\rangle - \left|\langle a(t)b^{\dagger}(t)\rangle\right|^2 = |f_2|^2(|\beta|^4 - |\alpha|^2|\beta|^2) - |g_3|^2|\alpha|^2|\beta|^2,\\
&\langle N_b(t)N_c(t)\rangle - \left|\langle b(t)c^{\dagger}(t)\rangle\right|^2 = 0, \quad (11)\\
&\langle N_a(t)N_c(t)\rangle - \left|\langle a(t)c^{\dagger}(t)\rangle\right|^2 = |f_2|^2|\beta|^2|\gamma|^2,
\end{aligned}$$

and,

$$\begin{aligned}
&\langle N_a(t)\rangle\langle N_b(t)\rangle - |\langle a(t)b(t)\rangle|^2 = |f_2|^2(|\beta|^4 + |\alpha|^2|\beta|^2),\\
&\langle N_b(t)\rangle\langle N_c(t)\rangle - |\langle b(t)c(t)\rangle|^2 = 0, \quad (12)\\
&\langle N_a(t)\rangle\langle N_c(t)\rangle - |\langle a(t)c(t)\rangle|^2 = |f_2|^2|\beta|^2|\gamma|^2.
\end{aligned}$$

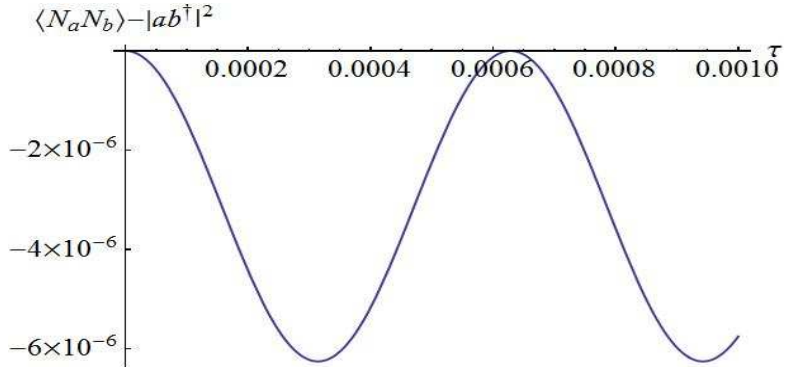

Figure 3. Intermodal entanglement using HZ-1 criterion for atomic-excited molecular modes for $\alpha = \beta = 5, \gamma = 100, \delta = 10\ MHz, \omega = \epsilon = 1\ KHz.$

Above equations show that excited molecular-stable molecular (bc)and atomic-stable molecular (ac) modes are always separable. It also clear from the equation (12) that HZ-2 criterion cannot detect entanglement between atomic-excited molecular (ab) modes. We plot HZ-1 criterion for ab mode. As the HZ-1 & HZ-2 criteria are sufficient for entanglement detection [14], the negative region of Fig. 3 indicates entanglement between atomic and excited molecular modes.For two-mode atom-molecule BEC system, the molecular mode is entangled with the atomic mode [8]. Whereas for the three mode atom-molecule BEC system one molecular mode (excited molecular) is entangled with atomic mode while the other (stable molecular) is separable with the atomic mode. For quantum computation, quantum teleportation, quantum computing where entangled states are required, atomic and excited molecular modes can be used. For entanglement distribution where separable state is required [15], the stable molecular mode can be used.

5. Conclusions

We solve the Hamiltonian of a three-mode atom-molecule BECs system prepared by STIRAP. We solve the Heisenberg's equations of motion using Sen-Mandal technique. Employing the solutions we study the quantum dynamics of the system and entanglement between the modes. We find that the particle number in all three modes are oscillate with time but population change is insignificant. Entanglement is observed only between atomic and excited molecular mode.

Since one molecular mode is entangled and other is separable with the atomic mode, we can use the molecular mode as our requirement (entangle or separable). There are further scope of research of this system considering intra-mode interactions along with the inter-mode interactions.

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