Journal of Physical Sciences, Vol. 21, 2016, 73-90
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/journal
Published on 24 December 2016

The Index of Coefficient of Variation for Ranking Fuzzy Numbers

Majid Mousavi

Department of Chemistry, Azad University of Shahroud, Shahroud, Iran
E-mail: box.majid2008@gmail.com

Received 10 August 2016; accepted 1 November 2016

ABSTRACT

In this paper, we have also presented the *CV* index to improve Lee and Li's [11] method. Lee and Li rank fuzzy numbers based on two different criteria, namely, the fuzzy mean and the fuzzy spread of the fuzzy numbers, and they pointed out that human intuition would favor a fuzzy number with the following characteristics: When higher mean value and at the same time higher spread/or lower mean value and at the same time lower spread is present it is not easy to compare its orderings clearly. Therefore, we can efficiently use *CV* index to rank its ordering, and the *CV* criterion is ranked higher with smaller *CV* value. Many ranking methods have been proposed so far. However, there is yet no method that can always give a satisfactory solution to every situation; some are counterintuitive, not discriminating; some use only the local information of fuzzy values; some produce different rankings for the same situation. For overcoming the above problems, we propose a new method for ranking fuzzy numbers by distance method. Our method is based on calculating the centroid point, where the distance means from original point to the centroid point $(\tilde{x}, \tilde{y})$, and the $\tilde{x}$ index is the same as Murakami et al.'s [1] $\tilde{x}$. However, the $\tilde{y}$ index is integrated from the inverse functions of an LR-type fuzzy number.

Thus, we use ranking function $R(\tilde{A}) = \sqrt{(\tilde{x}^2 + \tilde{y}^2)}$ (distance index) as the order quantities in a vague environment. Our method can rank more than two fuzzy numbers simultaneously, and the fuzzy numbers need not be normal.

Keywords: Centroid point, Distance index, Normal (non-normal) fuzzy numbers, Ranking fuzzy numbers.

1. Introduction

In most of cases in our life, the data obtained for decision making are only approximation finding Comparative operation of triangular fuzzy sets.

In practical use, ranking fuzzy numbers is very important. For example, the concept of optimum or best choice to come true are completely based on ranking or comparison. Therefore, how to set the rank of fuzzy numbers has been one of the main problems. The concept of fuzzy numbers is presented by Jain [6] and Dubois and Prade [7]. To resolve the task of comparing fuzzy numbers, many authors have proposed fuzzy ranking methods which yield a totally ordered set or ranking. A review and comparison of these existing methods can be found in [8, 9, 12]. Some ranking methods assume that the membership function is normal, but in many cases, limitation to the normal membership function is not adequate.

Liou and Wang show [2], show that Chen's method [3] can give non-logical result. Further, many of them produce different rankings for the same problem. About centroid index, Yager's [4], $\tilde{x}_0$ index may be seen as a general formula for calculating both $\tilde{x}$ and $\tilde{y}$ given different functions $g(x)$. However, only when $g(x)=x$, Murakami et al.'s [1] $\tilde{x}$ can be calculated, and when $g(x)=\mu_{\tilde{A}}(x)/2$, Murakami et al.'s Yo can be derived. Thus, Murakami et al.'s method is not logically sound either. For overcoming the above-mentioned problems, we propose ranking of fuzzy numbers by distance method based on calculating the centroid point, where distance means from original point to the centroid point. Thus, we use the ranking function $R(\tilde{A})=\sqrt[n]{\left(\tilde{x}^2+\tilde{y}^2\right)}$ as order quantities in a vague environment, which can rank more than two fuzzy numbers simultaneously, and need not be normal. Furthermore, we also propose a coefficient of variation (*CV*) to improve Lee and Li's method [4]. Lee and Li's [11] method ranks fuzzy numbers based on two criteria, namely, the fuzzy mean and the fuzzy spread of the fuzzy numbers. Our *CV* index is defined as σ (standard error)/ μ (mean), which can solve Lee and Li's problem efficiently. In this way, our proposed method also can be easily solve problems of ranking fuzzy numbers. At last, we present three numerical examples to illustrate our proposed method, and compare with other ranking methods. The purpose of this paper is to present a comparison method for fuzzy numbers based on distance index and *CV* index.

2. Preliminaries

A fuzzy number is described as any fuzzy subset of the real line *R* with membership function f_A

1- f_A is a continuous mapping from R to the closed interval $[0,w]$, $0 \le w \le 1$,

2- $f_A(x)=0$ for all $x \in \left(-\infty,\underline{a}\right]$,

3- f_A is strictly increasing on $\left[\underline{a},a\right]$,

4- f_A is strictly decreasing on $\left[a,\bar{a}\right]$,

5- $f_A(x)=0$ for all $x \in \left[\bar{a},+\infty\right)$, where $0 \le w \le 1$, $\underline{a}$, a and $\bar{a}$ are real numbers.

If $w=1$, then the generalized fuzzy number *A* is called a normal triangular fuzzy number denote $A=\left(\underline{a},a,\bar{a}\right)$. If $a=b$ and $c=d$, then *A* is called a crisp interval.

The membership function f_A of A can be expressed as

$$f_A(x) = \begin{cases} f_A L(x) & \underline{a} \le x \le a \\ f_A R(x) & a \le x \le \bar{a} \end{cases} \tag{1}$$

In otherwise= 0

where $f_A L(x) : \left[\underline{a}, a\right] \to [0, w]$ and $f_A R(x) : \left[a, \bar{a}\right] \to [0, w]$.

The inverse function $f_A L(x)$ exists because $f_A L(x) : \left[\underline{a}, a\right] \to [0, w]$ is continuous and strictly increasing. Similarly, The inverse function of $f_A R(x)$ exists because $f_A R(x) : \left[a, \bar{a}\right] \to [0, w]$ is continuous and strictly increasing. $f_A^{-1} L(x)$ and $f_A^{-1} R(x)$ are continuous on *[0,w]* that means both $\int_0^w f_A^{-1} L(x)$ and $\int_0^w f_A^{-1} R(x)$ exist.

3. Ranking fuzzy numbers by distance method based on calculating centroid point

Method of ranking with centroid index finds the geometric center of a fuzzy numbers, $\tilde{A}$. Each geometric center corresponds to an $\tilde{x}$ value on the horizontal axis and an $\tilde{y}$ value on the vertical axis. Yager's [4] method calculates for each fuzzy number only Y value, and Murakami, Maeda, and Imamura's [1] method calculates both $\tilde{x}$ and $\tilde{y}$ for each fuzzy number. However, in Murakami et al.'s example, all values are same. Thus, the $\tilde{x}$ value seems to be the only rational index for comparing fuzzy numbers.

From Fig. 1, $\tilde{x}$ an value on the horizontal axis is the most important index for ranking fuzzy numbers. However, a $\tilde{y}$ value on the vertical axis is an aid index, only in special cases the value is an important index for ranking fuzzy numbers (such as all $\tilde{x}$ values are equal or left and right spreads are same for all fuzzy numbers).

For satisfying the above condition, and overcoming Murakami et al.'s [1] example, we propose ranking fuzzy numbers by distance method based on calculating both $\tilde{x}$ and $\tilde{y}$ values. Our method can be introduced as follows:

(1): The inverse function of $f_A L(x)$ exists because $f_A L(x) : \left[\underline{a}, a\right] \to [0, w]$ is continuous and strictly increasing. Similarly, The inverse function of $f_A R(x)$ exists because $f_A R(x) : \left[a, \bar{a}\right] \to [0, w]$ is continuous and strictly increasing. $f_A^{-1} L(x)$

and $f_A^{-1}R(x)$ are continuous on *[0,w]* that means both $\int_0^w f_A^{-1}L(x)$ and $\int_0^w f_A^{-1}R(x)$ exist. Let $A=\left(\underline{a},a,\bar{a}\right)$, $\underline{a}\le a\le\bar{a}$ be a fuzzy set on $R=(-\infty,+\infty)$. It is called a triangular fuzzy number, if its membership function is

$$f_A(x)=\begin{cases}\dfrac{x-\underline{a}}{a-\underline{a}} & \underline{a}\le x\le a\\[2ex] \dfrac{\bar{a}-x}{\bar{a}-a} & a\le x\le \bar{a}\end{cases}\qquad (2)$$

In otherwise = 0.

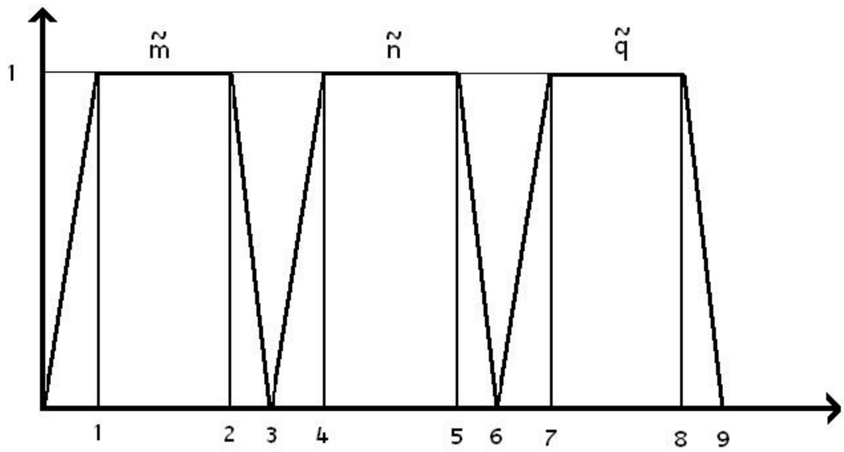

Figure 1. The ordering of fuzzy numbers is $\tilde{m}<\tilde{n}<\tilde{q}$

The inverse functions of $f_AL(x)$ and $f_AR(x)$ are $f_A^{-1}L(x)=\underline{a}+\left(a-\underline{a}\right)y$ and $f_A^{-1}R(x)=\bar{a}+\left(a-\bar{a}\right)y$.

(2) The centroid point of a fuzzy number corresponds to a $\tilde{x}$ on the horizontal axis and a $\tilde{y}$ on the vertical axis. The centroid point $(\tilde{x},\tilde{y})$ for a fuzzy number A is defined as [8]

$$\tilde{x}(A)=\frac{w\left[\int_{\underline{a}}^{a}(xf_A L)dx+\int_{a}^{\bar{a}}(xf_A R)dx\right]}{\int_{\underline{a}}^{a}(xf_A L)dx+\int_{a}^{\bar{a}}(xf_A R)dx}\,(\text{triangular fuzzy number})$$

$$\tilde{x}(A)=\frac{\int_{L_a}^{\underline{a}}(xf_A L)dx+\int_{\underline{a}}^{\bar{a}}x\,dx+\int_{\bar{a}}^{R_a}(xf_A R)dx}{\int_{L_a}^{\underline{a}}(f_A L)dx+\int_{\underline{a}}^{\bar{a}}dx+\int_{\bar{a}}^{R_a}(f_A R)dx}\,(\text{trapezoidal fuzzy number}) \qquad (3)$$

$$\tilde{y}(A)=\frac{\int_{0}^{1}\left(yf_A{}^{-1}L\right)dy+\int_{0}^{1}\left(yf_A{}^{-1}R\right)dy}{\int_{0}^{1}\left(f_A{}^{-1}L\right)dy+\int_{0}^{1}\left(f_A{}^{-1}R\right)dy}$$

where $f_A L(x)$ and $f_A R(x)$ are the left and right membership functions of fuzzy number *A*, respectively. $f_A{}^{-1}L(x)$ and $f_A{}^{-1}R(x)$ are the inverse functions of $f_A L(x)$ and $f_A R(x)$ respectively.

(3) Calculation of ranking function: We propose the distance index between the centroid point $\tilde{x},\tilde{y}$ and original point, i.e.,

$$R(A)=\sqrt{\left(\tilde{x}^2+\tilde{y}^2\right)} \qquad (4)$$

(4) **Ranking fuzzy numbers**: For any fuzzy numbers $A_i, A_j \in U$, where $U=\{A_1, A_2, ..., A_n\}$ is a set of convex fuzzy numbers. The ranking fuzzy number has the following properties:

- If $R(A_i) < \mathrm{R}(A_j)$, then $A_i < A_j$,
- If $R(A_i) = \mathrm{R}(A_j)$, then $A_i = A_j$,
- If $R(A_i) > \mathrm{R}(A_j)$, then $A_i > A_j$,

Finally, for non-numbers, it is given by

$$f_c(x)=\begin{cases}\dfrac{w\left(x-\underline{a}\right)}{a-\underline{a}} & if \qquad \underline{a}\le x\le a\\[2ex] \dfrac{w\left(\bar{a}-x\right)}{\bar{a}-a} & if \qquad a\le x\le \bar{a}\end{cases} \tag{5}$$

In otherwise = 0.

The inverse functions of $f_A L(x)$ and $f_A R(x)$ are $f_A{}^{-1}L(x)=\dfrac{\underline{a}+\left(a-\underline{a}\right)y}{w}$ and $f_A{}^{-1}R(x)=\dfrac{\bar{a}+\left(a-\bar{a}\right)y}{w}$, where $y\in[0,w]$.

By Equation. (3),

$$\tilde{x}(A)=\frac{w\int_{\underline{a}}^{a}\left[x\left(x-\underline{a}\right)/a-\underline{a}\right]dx+w\int_{a}^{\bar{a}}\left[x\left(\bar{a}-x\right)/\bar{a}-a\right]dx}{w\int_{\underline{a}}^{a}\left[\left(x-\underline{a}\right)/a-\underline{a}\right]dx+w\int_{a}^{\bar{a}}\left[\left(\bar{a}-x\right)/\bar{a}-a\right]dx}$$

$$=\frac{\int_{\underline{a}}^{a}(xf_A L)dx+\int_{a}^{\bar{a}}(xf_A R)dx}{\int_{\underline{a}}^{a}(f_A L)dx+\int_{a}^{\bar{a}}(f_A R)dx} \tag{6}$$

The $\tilde{x}$ value is the same as for normal fuzzy numbers.

$$\int_0^1 f_C^{-1\,L}(wy)\,dy=\int_0^1\left[\underline{a}+\left(a-\underline{a}\right)wy\ /w\right]dy=\int_0^1 f_A^{-1\,L}(y)\,dy$$

$$\int_0^1 f_C^{-1\,R}(wy)\,dy=\int_0^1\left[\bar{a}+\left(a-\bar{a}\right)wy\ /w\right]dy=\int_0^1 f_A^{-1\,R}(y)\,dy$$

and

$$\int_0^1 (wy)f_C^{-1\,L}(wy)\,dy=w\int_0^1\left[y\,\underline{a}+\left(a-\underline{a}\right)wy^2/w\right]dy=\int_0^1 (y)f_A^{-1\,L}(y)\,dy$$

$$\int_0^1 (wy)f_C^{-1\,R}(wy)\,dy=w\int_0^1\left[y\,\bar{a}+\left(a-\bar{a}\right)wy^2/w\right]dy=\int_0^1 (y)f_A^{-1\,R}(y)\,dy$$

Therefore,

$$\tilde{y}(A)=\frac{w\left[\int_0^1 (y) f_A^{-1\ L}(y)dy+\int_a^{\bar{a}} (y) f_A^{-1\ R}(y)dy\right]}{\int_0^1 f_A^{-1\ L}(y)dy+\int_a^{\bar{a}} f_A^{-1\ R}(y)dy} \tag{7}$$

where *w* is the adjustment factor for the $\tilde{y}$ value.

Lee and Li's method [11]:

We need a method for building a crisp total ordering from fuzzy numbers. Many methods for ranking of fuzzy numbers have been suggested. Each method appears to have some advantages as well as disadvantages [10]. In this section, firstly, we summarize Lee and Li's [11] method. Then, we propose the CV index to improve Lee and Li's method.

Lee and Li proposed the use of generalized mean and standard deviation based on the probability measures of fuzzy events to rank fuzzy numbers. The method ranks fuzzy numbers based on two different criteria, namely, the fuzzy mean and the fuzzy spread of the fuzzy numbers. They assume two kinds of probability distributions for fuzzy events and derive corresponding indices as follows:

(i) Uniform distribution: $f\left(\tilde{A}\right)=1/\left|\tilde{A}\right|$ and $\tilde{A}\in U$.

Its mean $x_U\left(\tilde{A}\right)$ and standards deviation $\sigma_U\left(\tilde{A}\right)$ are defined as

$$x_U\left(\tilde{A}\right)=\int_{S(\tilde{A})} x\,\mu_{\tilde{A}}(x)dx \Big/ \int_{S(\tilde{A})} \mu_{\tilde{A}}(x)dx$$

$$\sigma_U\left(\tilde{A}\right)=\left[\left(\int_{S(\tilde{A})} x^2\mu_{\tilde{A}}(x)dx \Big/ \int_{S(\tilde{A})} \mu_{\tilde{A}}(x)dx\right)-\left(x_U\left(\tilde{A}\right)\right)^2\right]^{1/2} \tag{8}$$

where $S\left(\tilde{A}\right)$ is the support of fuzzy number $\tilde{A}$.

4. The index of coefficient of variation

From the concept of statistics, the standard deviation and mean value cannot be the sole basis for comparing two fuzzy numbers, respectively. Furthermore, according to Lee and Li [11], higher mean value and at the same time lower spread is ranked higher. However, when higher mean value and at the same time higher spread/or lower mean value and at the same time lower spread, it is not easy to compare the orderings clearly. Therefore, we propose an efficient index, that is, using the coefficient of variation to improve Lee and Li's method.

The coefficient of variation is a relative measure of dispersion. It relates the standard deviation and the mean by expressing the standard deviation as a percentage of the mean. It is defined as

$CV=\sigma$ (standard error)/ $|\mu|$ (mean) (9)

Where $\mu \neq 0$ and $\sigma > 0$.

For comparing fuzzy numbers by our CV index method, firstly, we can calculate mean value and standard deviation by Lee and Li's method. Finally, the fuzzy number with smaller CV is ranked higher.

The following example was presented to illustrate our CV index method, which can improve Lee and Li's [11] method efficiently.

Example 4.1. The two trapezoidal fuzzy numbers $B_1 = (5,7,9,11)$ and $B_2 = (6,7,9,10)$ shown in Figure 2, which can be expressed as follows:

$$f_{B_1}(x) = \begin{cases} \frac{1}{2}(x-5) & 5 \leq x \leq 7 \\ \frac{1}{2}(x-11) & 9 \leq x \leq 11 \end{cases}$$

$$f_{B_2}(x) = \begin{cases} x-6 & 6 \leq x \leq 7 \\ x-10 & 9 \leq x \leq 10 \end{cases}$$

From Equations. (VIII)-(VIV), we can calculate its mean value if, standard deviation σ and *CV* values, which are listed in Table 1.

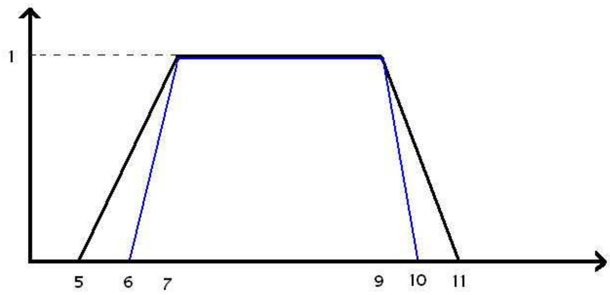

Figure 2. Two fuzzy numbers B_1, B_2.

Table 1. The $\tilde{x}$, σ and *CV* values for fuzzy numbers B_1, B_2.

	$\tilde{x}$	σ	CV
B_1	6.33	0.499	0.0788
B_2	7.33	0.499	0.068

However, Lee and Li's [11] criterion is higher mean value and at the same time lower spread. Clearly, we cannot easily compare its orderings by Lee and Li's method. Therefore, we can use our *CV* index to improve its shortcomings. From Table 1, we can easily rank its orderings by *CV* values, and the *CV* value of fuzzy number B_2 smaller than the fuzzy number B_1. Therefore, its ordering is $B_2 > B_1$.

5. Three numerical examples for illustrating our distance index

Example 5.1. In Fig. 3, three trapezoidal fuzzy numbers B_1, B_2, B_3 are shown.

$$f_{B_1}(x) = \begin{cases} \frac{1}{0.1}(x-0.2) & 0.2 \le x \le 0.3 \\ 1 & 0.3 \le x \le 0.5 \end{cases} = \begin{cases} \frac{1}{0.1}(0.6-x) & 0.5 < x \le 0.6 \\ 0 & otherwise \end{cases}$$

$$f_{B_2}(x) = \begin{cases} \frac{1}{0.1}(x-0.4) & 0.4 \le x \le 0.5 \\ 1 & 0.5 \le x \le 0.7 \end{cases} = \begin{cases} \frac{1}{0.1}(0.8-x) & 0.7 < x \le 0.8 \\ 0 & otherwise \end{cases}$$

$$f_{B_3}(x) = \begin{cases} \frac{1}{0.1}(x-0.6) & 0.6 \le x \le 0.7 \\ 1 & 0.7 \le x \le 0.9 \end{cases} = \begin{cases} \frac{1}{0.1}(1-x) & 0.9 < x \le 1 \\ 0 & otherwise \end{cases}$$

From step (1), we can derive the inverse function $f_{B_i}{}^{-1}L, f_{B_i}{}^{-1}R$ in Table 2.

By Equations. (III) and (IV), we can obtain the last results in Table 3. From Table 3, the ordering of fuzzy numbers is $B_1 < B_2 < B_3$. This is an example to show that in Murakami et al.'s [11] method, all y_{B_i} values are same. However, in our method, all y_{B_i} values are different.

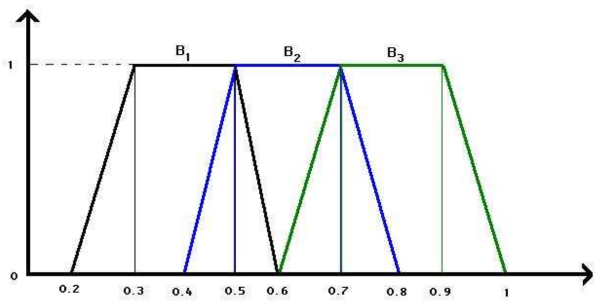

Figure 3. Three trapezoidal numbers B_1, B_2, B_3

Table 2. The inverse function $f_{B_i}{}^{-1}L, f_{B_i}{}^{-1}R$.

	$f_{B_i}{}^{-1}L$	$f_{B_i}{}^{-1}R$
B_1	0.2+0.1y	0.6-0.1y
B_2	0.4+0.1y	0.8-0.1y
B_3	0.6+0.1y	1-0.1y

$$\tilde{x}(B_1)=\frac{\int_{0.2}^{0.3} x\left(1/0.1(x-0.2)\right)dx+\int_{0.3}^{0.5} x\ dx+\int_{0.5}^{0.6} x\left(1/0.1(0.6-x)\right)dx}{\int_{0.2}^{0.3}\left(1/0.1(x-0.2)\right)dx+\int_{0.3}^{0.5} dx+\int_{0.5}^{0.6}\left(1/0.1(0.6-x)\right)dx}$$

$$=\frac{0.014+0.08+0.026}{0.05+0.2+0.05}=0.4,$$

$$\tilde{x}(B_2)=\frac{\int_{0.4}^{0.5} x\left(1/0.1(x-0.4)\right)dx+\int_{0.5}^{0.7} x\ dx+\int_{0.7}^{0.8} x\left(1/0.1(0.8-x)\right)dx}{\int_{0.4}^{0.5}\left(1/0.1(x-0.4)\right)dx+\int_{0.5}^{0.7} dx+\int_{0.7}^{0.8}\left(1/0.1(0.8-x)\right)dx}$$

$$=\frac{0.0233+0.12+0.037}{0.05+0.2+0.05}=0.601,$$

$$\tilde{x}(B_3)=\frac{\int_{0.6}^{0.7} x\left(1/0.1(x-0.6)\right)dx+\int_{0.7}^{0.9} x\ dx+\int_{0.9}^{1} x\left(1/0.1(1-x)\right)dx}{\int_{0.6}^{0.7}\left(1/0.1(x-0.6)\right)dx+\int_{0.7}^{0.9} dx+\int_{0.9}^{1}\left(1/0.1(1-x)\right)dx}$$

$$=\frac{0.033+0.16+0.08}{0.05+0.2-0.04}=0.91$$

$$\tilde{y}(B_1)=\frac{\int_0^1 y(0.2+0.1y)\ dy+\int_0^1 y(0.6-0.1y)\ dy}{\int_0^1 (0.2+0.1y)\ dy+\int_0^1 (0.6-0.1y)\ dy}=\frac{0.1333+0.267}{0.25+0.55}=0.5003,$$

$$\tilde{y}(B_2)=\frac{\int_0^1 y(0.4+0.1y)\ dy+\int_0^1 y(0.8-0.1y)\ dy}{\int_0^1 (0.4+0.1y)\ dy+\int_0^1 (0.8-0.1y)\ dy}=\frac{0.2333+0.367}{0.45+0.75}=0.5002,$$

$$\tilde{y}(B_3)=\frac{\int_0^1 y(0.6+0.1y)\ dy+\int_0^1 y(0.8-0.1y)\ dy}{\int_0^1 (0.6+0.1y)\ dy+\int_0^1 (0.8-0.1y)\ dy}=\frac{0.3333+0.367}{0.65+0.75}=0.4287$$

Table 3. The centroid point (x_i, y_i) and $R(B_i)=\sqrt{x_i^2+y_i^2}$.

	x_i	y_i	$R(B_i)=\sqrt{x_i^2+y_i^2}$
B_1	0.4	0.5003	0.0016
B_2	0.601	0.5002	0.0037
B_3	0.91	0.4287	0.0102

Example 5.2. In Fig. 4, three trapezoidal fuzzy numbers $B_1=(1,2,4,5;1)$ non-normal trapezoidal fuzzy number $B_2=(1,2,4,5;0.9)$, and triangular number $C_1=(4,7,8;1)$, non-normal triangular fuzzy numbers $C_2=(5,7,8;0.7)$ and $C_3=(6,7,8;0.5)$ are shown.

$$f_{B_1}(x)=\begin{cases} x-1 & 1\le x\le 2 \\ 1 & 2\le x\le 4 \end{cases}=\begin{cases} 5-x & 4<x\le 5 \\ 0 & otherwise \end{cases}$$

$$f_{B_2}(x)=\begin{cases} 0.9(x-1) & 1\le x\le 2 \\ 0.9 & 2\le x\le 4 \end{cases}=\begin{cases} 0.9(5-x) & 4<x\le 5 \\ 0 & otherwise \end{cases}$$

$$f_{C_1}(x)=\begin{cases} \frac{x-4}{3} & 4\le x\le 7 \\ 1 & x=7 \end{cases}=\begin{cases} 8-x & 7<x\le 8 \\ 0 & otherwise \end{cases}$$

$$f_{C_2}(x)=\begin{cases}0.7\left(\dfrac{x-5}{2}\right) & 5\le x\le 7\\ 0.7 & x=7\end{cases}=\begin{cases}0.7(8-x) & 7<x\le 8\\ 0 & otherwise\end{cases}$$

$$f_{C_3}(x)=\begin{cases}0.5(x-6) & 6\le x\le 7\\ 0.5 & x=7\end{cases}=\begin{cases}0.5(8-x) & 7<x\le 8\\ 0 & otherwise\end{cases}$$

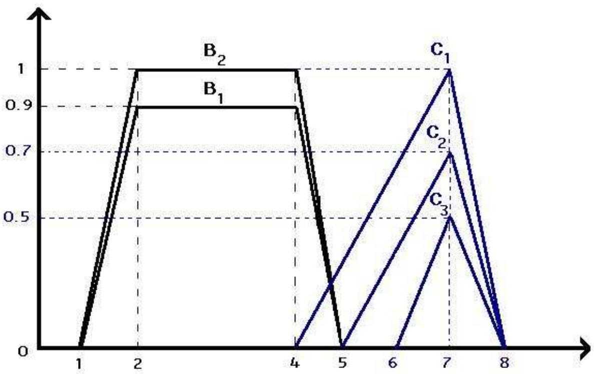

Figure 4. Five fuzzy numbers B_1, B_2 and C_1, C_2, C_3.

From step (1), we can derive the inverse function $f_{B_i}{}^{-1}L, f_{B_i}{}^{-1}R$ and $f_{C_i}{}^{-1}L, f_{C_i}{}^{-1}R$ in Table 4. By Equations. (III) and (IV), we can obtain the last results in Table 4. From Table 3, the ordering of fuzzy numbers is $B_1 < B_2 < C_1 < C_2 < C_3$. This is an example to show that in Lion and Wang's [2] method $B_1 = B_2$, all $\alpha \in [0,1]$. However, our method can rank its ordering.

Table 4. The inverse function $f_{B_i}{}^{-1}L, f_{C_i}{}^{-1}L$ and $f_{B_i}{}^{-1}R, f_{C_i}{}^{-1}R$.

	$f_{B_i}{}^{-1}L, f_{C_i}{}^{-1}L$	$f_{B_i}{}^{-1}R, f_{C_i}{}^{-1}R$
B_1	1+y	5-y
B_2	1+1/0.9y	5-1/0.9y
C_1	4+3y	8-y
C_2	5+2/0.7y	8-1/0.7y
C_3	6+1/0.5y	8-1/0.6y

$$\tilde{x}(B_1)=\frac{\int_1^2 x(x-1)\,dx+\int_2^4 x\,dx+\int_4^5 x(5-x)\,dx}{\int_1^2 (x-1)\,dx+\int_2^4 dx+\int_4^5 (5-x)\,dx}=\frac{0.83+6+2.17}{0.5+2+0.5}=0.3,$$

$$\tilde{x}(B_2)=\frac{\int_1^2 0.9(x(x-1))\,dx+\int_2^4 0.9x\,dx+\int_4^5 0.9(x(5-x))\,dx}{\int_1^2 0.9(x-1)\,dx+\int_2^4 0.9dx+\int_4^5 0.9(5-x)\,dx}=\frac{0.747+5.4+1.953}{0.45+1.8+0.45}=3,$$

$$\tilde{x}(C_1)=\frac{\int_4^7 1/3(x(x-4))\,dx+7+\int_7^8 x(8-x)\,dx}{\int_4^7 1/3(x-4)\,dx+1+\int_7^8 (8-x)\,dx}=\frac{9+7+3.67}{4.5+1+0.5}=3.27,$$

$$\tilde{x}(C_2)=\frac{\int_5^7 0.7/2(x(x-5))\,dx+7\times 0.7+\int_7^8 0.7(x(8-x))\,dx}{\int_5^7 0.7/2(x-5)\,dx+0.7+\int_7^8 0.7(8-x)\,dx}=\frac{4.4345+0.49+2.569}{0.7+0.7+0.35}=4.282,$$

$$\tilde{x}(C_3)=\frac{\int_6^7 0.5(x(x-6))\,dx+7\times 0.5+\int_7^8 0.5(x(8-x))\,dx}{\int_6^7 0.5(x-6)\,dx+0.5+\int_7^8 0.5(8-x)\,dx}=\frac{1.665+3.5+1.835}{0.25+0.5+0.25}=7,$$

$$\tilde{y}(B_1)=\frac{w\left[\int_0^1 y(1+y)\ dy+\int_0^1 y(5-y)\ dy\right]}{\int_0^1 (1+y)\ dy+\int_0^1 (5-y)\ dy}=\frac{1[0.833+2.166]}{1.5+4.5}=0.4998,$$

$$\tilde{y}(B_2)=\frac{w\left[\int_0^1 y(1+y)\ dy+\int_0^1 y(5-y)\ dy\right]}{\int_0^1 (1+y)\ dy+\int_0^1 (5-y)\ dy}=\frac{0.9[0.833+2.166]}{1.5+4.5}=0.4498,$$

$$\tilde{y}(C_1)=\frac{w\left[\int_0^1 y(4+3y)\ dy+\int_0^1 y(8-y)\ dy\right]}{\int_0^1 (4+3y)\ dy+\int_0^1 (8-y)\ dy}=\frac{1[3+3.66]}{5.5+7.5}=0.5123,$$

$$\tilde{y}(C_2)=\frac{w\left[\int_0^1 y(5+2y)\ dy+\int_0^1 y(8-y)\ dy\right]}{\int_0^1 (5+2y)\ dy+\int_0^1 (8-y)\ dy}=\frac{0.7[3.166+3.66]}{6+7.5}=0.3539,$$

$$\tilde{y}(C_3)=\frac{w\left[\int_0^1 y(6+y)\ dy+\int_0^1 y(8-y)\ dy\right]}{\int_0^1 (6+y)\ dy+\int_0^1 (8-y)\ dy}=\frac{0.5[3.33+3.66]}{6.5+7.5}=0.2496$$

Table 5. The centroid point (x_i, y_i) and $R(X)=\sqrt{x_i^2+y_i^2}$.

	x_i	y_i	$R(X)=\sqrt{x_i^2+y_i^2}$
B_1	3	0.4998	3.041
B_2	3	0.4498	3.033
C_1	3.27	0.5123	3.309
C_2	4.282	0.3539	4.296
C_3	7	0.2496	7.004

Example 5.3. In Fig. 5, With trapezoidal fuzzy numbers $B_1=(0.1,0.2,0.5,0.9;1)$ $B_2=(0.1,0.4,0.6,0.9;1)$ and $B_3=(0.1,0.5,0.8,0.9;1)$. Their membership functions are defined as

$$f_{B_1}(x)=\begin{cases}\frac{1}{0.1}(x-0.1) & 0.1\le x\le 0.2\\ 1 & 0.2\le x\le 0.5\end{cases}=\begin{cases}\frac{1}{0.4}(0.9-x) & 0.5<x\le 0.9\\ 0 & othewise\end{cases}$$

$$f_{B_2}(x)=\begin{cases}\frac{1}{0.3}(x-0.1) & 0.1\le x\le 0.4\\ 1 & 0.4\le x\le 0.6\end{cases}=\begin{cases}\frac{1}{0.3}(0.9-x) & 0.6<x\le 0.9\\ 0 & othewise\end{cases}$$

$$f_{B_3}(x)=\begin{cases}\dfrac{1}{0.4}(x-0.1) & 0.1\le x\le 0.5\\ 1 & 0.5\le x\le 0.8\end{cases}=\begin{cases}\dfrac{1}{0.1}(0.9-x) & 0.8<x\le 0.9\\ 0 & otherwise\end{cases}$$

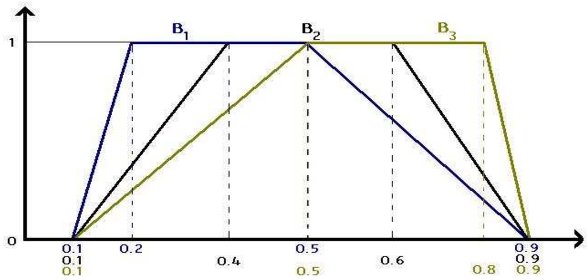

Figure 5. Three trapezoidal fuzzy numbers B_1, B_2 and B_3.

From step (1), we can derive the inverse function $f_{B_i}{}^{-1}L, f_{B_i}{}^{-1}R$ in Table 6.
By Equations. (III) and (IV), we can obtain the last results in Table 7. From Table 4, the ordering of fuzzy numbers is $B_1 < B_2 < B_3$. According to Lee and Li [11], a fuzzy number with larger mean and smaller spread should be ranked higher. Due to this, all fuzzy sets have the same spread. Therefore, we can directly rank their ordering by $\tilde{x}_i$.

Table 6. The inverse function $f_{B_i}{}^{-1}L, f_{B_i}{}^{-1}R$.

	$f_{B_i}{}^{-1}L$	$f_{B_i}{}^{-1}R$
B_1	0.1+0.1y	0.9-0.4y
B_2	0.1+0.3y	0.9-0.3y
B_3	0.1+0.4y	0.9-0.1y

$$\tilde{x}(B_1)=\frac{\int_{0.1}^{0.2} x\left(1/0.1(x-0.1)\right)\,dx+\int_{0.2}^{0.5} x\,dx+\int_{0.5}^{0.9} x\left(1/0.4(0.9-x)\right)\,dx}{\int_{0.1}^{0.2}\left(1/0.1(x-0.1)\right)\,dx+\int_{0.2}^{0.5} dx+\int_{0.5}^{0.9}\left(1/0.4(0.9-x)\right)\,dx}=\frac{0.008+0.105+0.1265}{0.05+0.3+0.2}=0.4354,$$

$$\tilde{x}(B_2)=\frac{\int_{0.1}^{0.4} x\left(1/0.3(x-0.1)\right)\,dx+\int_{0.4}^{0.6} x\;dx+\int_{0.6}^{0.9} x\left(1/0.3(0.9-x)\right)\,dx}{\int_{0.1}^{0.4}\left(1/0.3(x-0.1)\right)\,dx+\int_{0.4}^{0.6} dx+\int_{0.6}^{0.9}\left(1/0.3(0.9-x)\right)\,dx}=\frac{0.045+0.1+0.105}{0.15+0.2+0.15}=0.45,$$

$$\tilde{x}(B_3)=\frac{\int_{0.1}^{0.5} x\left(1/0.4(x-0.1)\right)\,dx+\int_{0.5}^{0.8} x\;dx+\int_{0.8}^{0.9} x\left(1/0.1(0.9-x)\right)\,dx}{\int_{0.1}^{0.5}\left(1/0.4(x-0.1)\right)\,dx+\int_{0.5}^{0.8} dx+\int_{0.8}^{0.9}\left(1/0.1(0.9-x)\right)\,dx}=\frac{0.07325+0.195+0.041}{0.2+0.3+0.5}=0.5622$$

$$\tilde{y}(B_1)=\frac{\int_{0}^{1} y\left(0.1+0.1y\right)\,dy+\int_{0}^{1} y\left(0.9-0.4y\right)\,dy}{\int_{0}^{1}\left(0.1+0.1y\right)\,dy+\int_{0}^{1}\left(0.9-0.4y\right)\,dy}=\frac{0.0833+0.3166}{0.15+0.7}=0.4704,$$

$$\tilde{y}(B_2)=\frac{\int_{0}^{1} y\left(0.1+0.3y\right)\,dy+\int_{0}^{1} y\left(0.9-0.3y\right)\,dy}{\int_{0}^{1}\left(0.1+0.3y\right)\,dy+\int_{0}^{1}\left(0.9-0.3y\right)\,dy}=\frac{0.15+0.35}{0.25+0.75}=0.5,$$

$$\tilde{y}(B_3)=\frac{\int_{0}^{1} y\left(0.1+0.4y\right)\,dy+\int_{0}^{1} y\left(0.9-0.1y\right)\,dy}{\int_{0}^{1}\left(0.1+0.4y\right)\,dy+\int_{0}^{1}\left(0.9-0.1y\right)\,dy}=\frac{0.1833+0.4166}{0.3+0.85}=0.5216$$

Table 7. The centroid point (x_i, y_i) and $R(B_i)=\sqrt{x_i^{\,2}+y_i^{\,2}}$.

	x_i	y_i	$R(B_i)=\sqrt{x_i^{\,2}+y_i^{\,2}}$
B_1	0.4354	0.4704	0.4108
B_2	0.5	0.5	0.7071
B_3	0.5622	0.5216	0.7668

Yager's [4] $\tilde{x}_0$ index measures the general mean of fuzzy numbers. It is not surprising to see that the index alone provides very poor discrimination ability. Yager's [4] $\tilde{x}_0$ index may be seen as a general formula for calculating both $\tilde{x}_0$ and $\tilde{y}_0$ given different functions *g(x)*. However, only when *g(x) = x*, Murakami et al.'s [1] $\tilde{x}_0$ can be calculated, and when $g\left(x\right)=\frac{1}{2}\mu_{\tilde{A}}\left(x\right)$, Murakami et al.'s [1] $\tilde{y}_0$ can be derived. Thus Murakami et al.'s [1] method is not logically sound either.

We have proposed a new approach for ranking fuzzy numbers, which are based on calculating both $\tilde{x}_0$ and $\tilde{y}_0$ values and distance index.

Further, we have also presented the *CV* index to improve Lee and Li's method. Therefore, we can efficiently use *CV* index to rank its ordering, and the *CV* criterion is ranked higher with smaller *CV* value.

6. Conclusion

In this paper, We have also presented the *CV* index to improve Lee and Li's [11] method. In this way, our proposed method also can be easily solve problems of ranking fuzzy numbers At last, we present three numerical examples to illustrate our proposed method, and compare with other ranking methods. The purpose of this paper is to present a comparison method for fuzzy numbers based on distance index and *CV* index.

REFERENCES

1. S.Murakami, S.Maeda and S.Imamura, Fuzzy decision analysis on the development of centralized regional energy control; system, *IFAC Syrup. on Fuzzy Inform. Knowledge Representation and Decision Anal.* (1983) 363-368.
2. T.S.Liou and M.J.J.Wang, Ranking fuzzy numbers with integral value, *Fuzzy Sets and Systems,* 50 (1992) 247-255.
3. S.H.Chen, Ranking fuzzy numbers with maximizing set and minimizing set, *Fuzzy Sets and Systems,* 17 (1985) 113-129.
4. R.R.Yager, A procedure for ordering fuzzy subsets of the unit interval, *Inform. Sci.,* 24 (1981) 143-161.
5. E.S.Lee and R.L.Li, Comparison of fuzzy numbers based on the probability measure of fuzzy events, *Comput. Math. Appl.,* 15 (1988) 887-896.
6. R.Jain, A procedure for multi-aspect decision making using fuzzy sets, *Internat. J. Systems Sci.,* 8 (1978) 1-7.
7. D.Dubois and H.Prade, Operations on fuzzy numbers, *lnternat. J. Systems Sci.,* 9 (1978) 631-626.
8. S.J.Chen and C.L.Hwang, Fuzzy multiple attribute decision making methods and applications, *Lecture Notes in Economics and Mathematical Systems (Springer, New York, 1992).*
9. Y.Deng, Z.Zhenfu and L. Qi, "Ranking Fuzzy Numbers with an Area Method using Radius of Gyration", *Computers and Mathematics with Applications* 51 (2006) 1127-1136.
10. E.S.Lee and R.L.Li, Comparison of fuzzy numbers based on the probability measure of fuzzy events, *Comput. Math. Appl.,* 15 (1988) 887-896.

11. Y.J.Wang and H.S.Lee, The revised method of ranking fuzzy numbers with an area between the centroid and original points, *Computers and Mathematics with Applications*, 55 (2008) 2033-2042.